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MAIRA GIOVANA DE SOUZA

GENERAL RELATIVITY AT HIGH SCHOOL: A STUDENTS' MENTAL REPRESENTATIONS AND CONCEPTIONS STUDY

Canoas, 2024

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Thesis presented at the Science and Mathematics Teaching Graduate Program from Lutheran University of Brazil to obtain the title of Doctor in Science and Mathematics Teaching.

Supervisor: Prof. PhD Agostinho Serrano de Andrade Neto (01/01/21 to 16/09/24)

Supervisor: Prof. PhD Agostinho Iaqchan Ryokiti Homa (from 17/09/24)

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“Somewhere, something incredible
is waiting to be known”.

Carl Sagan

RESUMO

Esta tese de doutorado investiga as representações mentais e as concepções de estudantes do Ensino Médio sobre a Teoria da Relatividade, com foco na Relatividade Geral (TRG). A pesquisa, fundamentada na Teoria da Mediação Cognitiva em Rede (TMC), explora como diferentes níveis de mediação externa influenciam o desenvolvimento de representações mentais e da compreensão dos alunos sobre conceitos complexos e contraintuitivos, como espaço-tempo curvo e dilatação temporal gravitacional. A pesquisa se estrutura em três experimentos: o Experimento 1, derivado da pesquisa de mestrado, analisa as representações mentais dos estudantes sobre a Relatividade Especial, servindo como base para os experimentos subsequentes. Os Experimentos 2 e 3, realizados em formato de minicurso extracurricular, investigam como os alunos constroem representações mentais para conceitos da TRG. Os dados coletados por meio de pré-testes, pós-testes, entrevistas, análise gestual e materiais produzidos pelos estudantes revelam uma forte conexão entre a consistência das representações mentais e a compreensão conceitual. Estudantes com representações mentais pictóricas bem definidas, frequentemente desenvolvidas a partir de interações com simulações computacionais e o modelo da malha, demonstraram melhor compreensão de espaço-tempo curvo. Por outro lado, a dilatação temporal foi mais frequentemente compreendida a partir de representações proposicionais, formadas através de discussões em sala de aula e exercícios. A pesquisa evidencia a influência de diferentes níveis de mediação externa na construção de *drivers*, que se manifestam como representações mentais. Simulações computacionais, o modelo da malha, discussões em grupo, explicações da professora, referências culturais, como o filme Interestelar, e ferramentas de inteligência artificial generativa contribuíram de forma variada para o desenvolvimento da compreensão dos alunos. Conclui-se que uma abordagem multimodal, que reconheça o valor de recursos diversificados trabalhando colaborativamente, é fundamental para o aprendizado eficaz da TRG. A TMC se mostra um referencial teórico eficaz para analisar e compreender a interação dinâmica de mediações externas na formação de representações internas, fornecendo *insights* valiosos para o planejamento de currículos e práticas pedagógicas e a integração de ferramentas de IA no ensino de física.

Palavras-chaves: ensino de física; Teoria da Relatividade; representações mentais; mediação externa; análise gestual.

ABSTRACT

This doctoral thesis investigates the mental representations and conceptions of high school students about Relativity Theory, focusing on the General Relativity (GR). The research, grounded in the Cognitive Mediation Networks Theory (CMNT), explores how different levels of external mediation influence the development of mental representations and students' understanding of complex and counterintuitive concepts, such as curved spacetime and gravitational time dilation. The research consists of three experiments: Cohort 1, derived from the master's research, analyses students' mental representations of Special Relativity, serving as a basis for the subsequent experiments. Cohort 2 and 3, conducted in the format of extracurricular mini courses, investigate how students construct mental representations of GR concepts. The data collected through pre-tests, post-tests, interviews, gesture analysis, and student-produced materials reveal a strong connection between the consistency of mental representations and conceptual understanding. Students with well-defined pictorial mental representations, often developed through interactions with computer simulations and the rubber-sheet model, demonstrated better understanding of curved spacetime. On the other hand, gravitational time dilation was more frequently understood through propositional representations, formed through classroom discussions and exercises. The research shows the influence of different levels of external mediation on the construction of *drivers*, which manifest as mental representations. Computer simulations, the rubber-sheet model, group discussions, teacher explanations, cultural references such as the movie Interstellar, and Generative Artificial Intelligence tools contributed variably to students' understanding. The conclusion is that a multimodal approach, recognizing the value of diverse resources working collaboratively, is essential for effective learning of GR. The CMNT proves to be an effective theoretical reference for analysing and understanding the dynamic interaction of external mediations in the formation of internal representations, providing valuable insights for curriculum planning and pedagogical practices, and the integration of AI tools in physics education.

Keywords: physics teaching; Relativity Theory; mental representations; external mediation; gestural analysis.

RESUMEN

Esta tesis doctoral investiga las representaciones mentales y concepciones de los estudiantes de educación media sobre la Teoría de la Relatividad, con un enfoque en la teoría de la Relatividad General (TRG). La investigación, fundamentada en la Teoría de la Mediación Cognitiva en Red (TMC), explora cómo diferentes niveles de mediación externa influyen en el desarrollo de representaciones mentales y la comprensión de los estudiantes sobre conceptos complejos y contraintuitivos, como el espacio-tiempo curvo y la dilatación temporal gravitacional. La investigación se estructura en tres experimentos: el Experimento 1, derivado de la investigación de maestría, analiza las representaciones mentales de los estudiantes sobre la Relatividad Especial, sirviendo como base para los experimentos subsiguientes. Los Experimentos 2 y 3, realizados en formato de minicursos extracurriculares, investigan cómo los estudiantes construyen representaciones mentales para conceptos de la TRG. Los datos recopilados a través de pruebas previas, pruebas posteriores, entrevistas, análisis de gestos y materiales producidos por los estudiantes revelan una conexión fuerte entre la consistencia de las representaciones mentales y la comprensión conceptual. Los estudiantes con representaciones mentales pictóricas bien definidas, a menudo desarrolladas a través de interacciones con simulaciones computacionales y el modelo de malla, demostraron una comprensión mejor del espacio-tiempo curvo. Por otro lado, la dilatación temporal fue más frecuentemente comprendida a través de representaciones proposicionales, formadas a través de discusiones en clase y ejercicios. La investigación muestra la influencia de diferentes niveles de mediación externa en la construcción de *drivers*, que se manifiestan como representaciones mentales. Simulaciones computacionales, el modelo de malla, discusiones en grupo, explicaciones de la profesora, referencias culturales, como la película Interestelar, y herramientas de inteligencia artificial generativa contribuyeron de manera variable al desarrollo de la comprensión de los estudiantes. Se concluye que un enfoque multimodal, que reconozca el valor de recursos diversificados trabajando colaborativamente, es esencial para el aprendizaje efectivo de la TRG. La TMC se muestra como un referencial teórico efectivo para analizar y comprender la interacción dinámica de mediaciones externas en la formación de representaciones internas, proporcionando valiosos *insights* para el planeamiento de currículos y prácticas pedagógicas y la integración de herramientas de IA en la enseñanza de la física.

Palabras clave: enseñanza de física; Teoría de la Relatividad; representaciones mentales; mediación externa; análisis gestual.

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PUBLICATIONS LIST

During the enrolment for the doctoral degree, the following works were developed.

- Published papers:

DE SOUZA, M. G.; SERRANO, A. The impact of the usage of hypercultural mediation in the teaching of Special Relativity in high school in Brazil. **Acta Scientiae**, v. 22, n. 4, p. 2-27, 2020. https://doi.org/10.17648/ACTA.SCIENTIAE.5875

DE SOUZA, M. G.; SERRANO, A. Resenha–Teaching Einsteinian physics in schools: an essential guide for teachers in training and practice de Magdalena Kersting e David Blair. **Revista Brasileira de Ensino de Física**, v. 45, e20220250, 2023. https://doi.org/10.1590/1806-9126-RBEF-2022-0250

DE SOUZA, M. G.; WON, M.; TREAGUST, D.; SERRANO, A. Visualising relativity: assessing high school students' understanding of complex physics concepts through AI-generated images. **Physics Education**, v. 59, n. 2, p. 025018, 2024. https://doi.org.10.1088/1361-6552/ad1e71

DOS ANJOS, J. R.; DE SOUZA, M. G.; SERRANO, A.; SOUZA, B. C. An analysis of the generative AI use as analyst in qualitative research in science education. **Revista Pesquisa Qualitativa**, v. 12, n. 30, p. 01-29, 2024. https://doi.org/10.33361/RPQ.2024.v.12.n.30.724

DOS ANJOS, J. R.; DE SOUZA, M. G.; PRODANOV, T. S.; SERRANO, A. Disruptive Education: Integrating ChatGPT into an Active Methodology for Teaching Sciences and Mathematics. **Acta Scientiae**, v. 26, n. 1, p. 334-369, 2024. https://doi.org/10.17648/acta.scientiae.7993

DE SOUZA, M. G.; SERRANO, A.; TREAGUST, D. Exploring the relationship between mental representations and conceptual understanding of Special Relativity by high school students. **Research in Science and Technological Education**, v. 42, 2024. https://doi.org/10.1080/02635143.2024.2446801

- Accepted papers:

DE SOUZA, M. G. et al. Introduzindo física einsteiniana nas escolas: a abordagem Einstein-First. [Introducing Einsteinian physics at schools: the Einstein-First approach]. **Revista Física na Escola**.

- Papers under review:

DE SOUZA, M. G.; SERRANO, A.; TREAGUST, D.; WON, M. Conceptions of Curved Spacetime: relating students' mental representations and understanding of General Relativity. **Research in Science Education.**

DE SOUZA, M. G.; SERRANO, A.; TREAGUST, D. External mediation educational resources for teaching General Relativity: a systematic review. **Journal of Turkish Science Education.**

- Conference papers:

DE SOUZA, M. G.; SERRANO, A., 2021, Braga, Portugal. **ESERA Conference.** Using different mediations to foster the development of relativistic zone in conceptual profile of reference frame. CIEC, University of Minho, 2022. 8. http://dx.doi.org/10.5281/zenodo.10700246

DE SOUZA, M. G.; ANJOS, J. R.; SERRANO, A., 2022, Maceió, Alagoas. **VIII CONEDU.** Representações mentais de relatividade especial: uma análise sob a ótica da teoria da codificação dual. [Special Relativity mental representations: an analysis under the lens of Dual Coding Theory]. Campina Grande: Realize Editora, 2022. 12. http://dx.doi.org/10.5281/zenodo.10700186

DE SOUZA, M. G.; SERRANO, A.; ANJOS, J. R., 2023, Cappadocia, Turkey. **ESERA Conference.** What are High School Students' Mental Representations about Relativity Theory? Nobel Bilimsel Eserler, 2024. 8. http://dx.doi.org/10.5281/zenodo.1375483

- Papers in production:

DE SOUZA, M. G.; SERRANO, A.; TREAGUST, D. Efficient chatbot communication: analysing students' interactions with AI chatbots in a relativity course.

- Presentations:

Tópicos de Física como mediadores entre a pesquisa e a sala de aula: luz, relatividade e exoplanetas. [Modern physics topics as mediators between the research and classroom: light, relativity and exoplanets]. Seminar, Lutheran University of Brazil, 2022.

Explorando o uso e potencial da IA no Ensino. [Exploring the use and potential of AI for teaching]. Seminar, Lutheran University of Brazil, 2023.

Possibilities for Teaching General Relativity. Workshop, STAWA Future Science, 2023.

What are high school students' mental representations about Relativity Theory? Poster presentation, School of Education, Curtin University, 2023.

Visualising relativity: assessing high school students' understanding of complex physics concepts through AI-generated images. IMPRESS Seminar, 2024.

What is going on inside their heads? An interview analysis methodology for science education research. Seminar, School of Education, Curtin University, 2024.

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1 INTRODUCTION

The Relativity Theory (RT) had a profound impact on the scientific community and society. With revolutionary ideas, Albert Einstein changed our understanding of space and time. These changes influenced the development of science after the consolidation of the theory, taking new and unforeseen directions (Beirão; Silva; Nunes; Campos, 2019). Although these new interpretations of reality do not have a direct and evident impact on daily life, they enabled and fostered technological advancements in the last century. Thus, all individuals ultimately experience the impacts of this revolutionary theory, whether within the scientific community or simply by using GPS (Global Positioning System) on their smartphones, for example (Boublil; Blair; Treagust, 2023).

Considering its relevance to society as a whole, consequently the Relativity Theory is of great importance in basic education, mainly secondary schools, albeit in a non-in-depth manner. It is essential that students have knowledge of the fundamental bases of current science and the theories that provide the most accurate current description of the universe (Kaur; Blair; Moschilla; Stannard *et al.*, 2017). The lack of such knowledge can posit a significant obstacle to the development of critical thinking and active insertion of these students in society (Choudhary; Foppoli; Kaur; Blair *et al.*, 2022).

However, the complexity of the ideas proposed by Albert Einstein is widely recognized (Kersting, 2019b; Mcinerney; Sutton, 2024). This inclination often leads educators to believe that topics like this cannot be addressed in the schools' classroom (Nicolau J.; Brockington; Sasseron, 2011). In fact, the difficulty that students, even at the university level, face with some aspects of the theory is recognized (Bandyopadhyay; Kumar, 2010; Kamphorst; Vollebregt; Savelsbergh; Van Joolingen, 2023; Steier; Kersting, 2019). However, on the other hand, the positive impact of inserting such topics in the classroom has been observed, and it has been found that even with difficulties, schools' students are capable of understanding the basic aspects of relativity (Kaur; Blair; Stannard; Treagust *et al.*, 2020; Ruggiero; Mattiello; Leone, 2021).

Therefore, investigations into the insertion of this topic in schools and the learning of relativity by students become crucial. Based on these results, it is possible to develop valuable strategies for addressing this fascinating theory in the classroom.

In this context, the present doctoral research is inserted. It is considered that the development of mental representations is crucial in learning scientific concepts (Kosslyn; Thompson; Ganis, 2006). Through them, students can reproduce phenomena and their effects mentally or access logical mental structures for their explanation, facilitating their understanding. This type of resource is even more necessary when dealing with phenomena that require great abstraction, such as those related to the Relativity Theory.

In this sense, activities addressing the Relativity Theory that stimulate the construction of mental representations were planned. For this, resources using different levels of external mediation were selected (Souza; Da Silva; Da Silva; Roazzi *et al.*, 2012; Souza; Serrano; Roazzi, 2024). These activities were developed with high school students to later analyse the development of mental representations of crucial concepts of the RT by them. More precisely, the aim is to investigate the relationship between these mental representations and the conceptual understanding of the theory by students.

Through results found in previous studies (De Souza, 2021; De Souza; Serrano, 2020), it was perceived a connection between the use of different external mediation resources and the construction of these mental representations. Since the master’s research focused on Special Relativity (SR), the doctoral research expanded the study to the context of General Relativity (GR).

2 RESEARCH: BASIC ASPECTS

This doctoral research is linked to the research topic “Information and Communication Technologies to Science and Mathematics Teaching” of the Science and Mathematics Teaching Graduate Program (PPGECIM/ULBRA) and has the theme of Modern Physics Teaching and the mental representations’ investigation developed, both propositional and pictorial.

2.1 RESEARCH QUESTION

The guiding problem of the research is, after the students’ interaction with different levels of external mediations, to investigate how their mental representations are developed in the process of understanding the key concepts of the Relativity Theory. The aim is to identify what types of representations are developed, whether propositional or pictorial, as well as the students’ conceptions regarding the theory. Thus, the research seeks to answer the following question:

How do students’ mental representations and conceptions of Relativity Theory develop?

2.2 OBJECTIVES

Here the general and specific objectives of this research are presented.

2.2.1 General Objective

The main objective of this doctoral research is to understand and analyse how high school students’ mental representations are developed, whether they are depictive or propositional, as well as their connection with students conceptual understanding regarding RT.

2.2.2 Specific Objectives

- Develop a teaching sequence addressing since Galilean Relativity to the General Relativity Theory;
- Discuss didactically the flaws in Galilean Relativity, demonstrating the need for the development of the Relativity Theory;
- Present the historical facts that led to the development of the Relativity Theory;

- Present conceptual models for Special Relativity within the different levels of mediation of the CMNT;
- Present conceptual models for General Relativity within the different levels of mediation of the CMNT;
- Identify and analyse the students' conceptions for understanding General Relativity;
- Identify and analyse the mental representations constructed by students for understanding General Relativity, whether propositional or pictorial.

2.3 RATIONALE

There is a growing disinterest among students in science, not only in Brazil but also globally (Dodd, 2024; Treagust; Won; Petersen; Wynne, 2015). When it comes to Physics, this aversion is even more pronounced, as it is often approached as a more complex extension of Mathematics. As a consequence, there is a lack of professionals in STEM careers (science, technology, engineering, and mathematics) with no prospects of change in the near future, as many young people opt not to pursue careers in this area (Gouw; Mota; Bizzo, 2016; Treagust; Won; Petersen; Wynne, 2015).

Among the possible reasons for students' disinterest, there is the outdated school system, which is far removed from their reality (Blandforf; Thorne, 2020; Santos; Ribeiro, 2020). Many advances in modern society have been made possible by the development of Modern and Contemporary Physics (MCP). Moreover, young people daily encounter technologies that incorporate introductory concepts of MCP (Assunção; Nascimento, 2019; Boublil; Blair; Treagust, 2023). Therefore, what is taught in schools often fails to explain a significant part of what students encounter in their daily lives.

This gap between school education and understanding of the modern world can lead to misconceptions that become barriers to the formation of scientifically literate citizens (Kersting; Blair, 2021). It has been observed that the performance in science of many OECD nations, including Brazil, has decreased between 2018 and 2022 (Oecd, 2023). Hence, aligning school instruction with the best scientific understanding of society can improve students' attitudes towards science, contributing to their scientific literacy.

The Brazilian National Common Curriculum Base (BNCC), regarding the Natural Sciences, emphasizes that basic education has a “commitment to the development of scientific literacy, which involves the ability to understand and interpret the world” and that students should develop the “ability to act in and on the world, essential for the full exercise of citizenship” (Brasil, 2018, p. 321).

Furthermore, the presence topics of Modern and Contemporary Physics, including the Relativity Theory, has increased in social media, instigating young people to learn more about these subjects. Recent events, such as the detection of gravitational waves (Abbott; Abbott; Abbott; Abernathy *et al.*, 2016), the 2020 Nobel Prize in Physics (Pinheiro, 2020) and the first images of black holes (Bower; Van Langlevende, 2022) have brought more visibility to the theory.

In this context, students generally show great interest in topics of Modern and Contemporary Physics (Choudhary; Foppoli; Kaur; Blair *et al.*, 2022), especially the Relativity Theory, given its connection to astronomy (Kersting, 2019b). It has been observed that, when studying relativity topics, students become more motivated to study physics (Vakarou; Stylos; Kotsis, 2024). Therefore, addressing these topics in the classroom can captivate and engage young people to learn science, making it more meaningful to them (Foppoli; Choudhary; Blair; Kaur *et al.*, 2019).

However, the Relativity Theory is a complex topic, and several challenges arise when addressing it in schools. Firstly, there is the issue of presenting the concepts appropriately to students, transposing scientific knowledge into teachable knowledge. Moreover, students generally have difficulties dealing with counterintuitive and abstract topics (Ruggiero; Mattiello; Leone, 2021; Velentzas; Halkia, 2013), such as curved spacetime, for example (Mcinerney; Sutton, 2024). Furthermore, relativistic phenomena are hardly observable in daily life (Bandyopadhyay; Kumar, 2010), making it challenging for students to visualize them (Steier; Kersting, 2019; Yavaş; Kızılcık, 2016).

As a result, the Relativity Theory and other Modern and Contemporary Physics topics often are not addressed in the classroom (Kaur; Blair; Stannard; Treagust *et al.*, 2020; Pitts; Venville; Blair; Zadnik, 2014; Siqueira; Montanha; Batista; Pietrocola, 2018). Another important aspect to be considered is the Physics teachers’ education. For changes in the school setting to be effective, the professional development of teachers is essential (Aldridge; Mclure, 2023; Robinson; Bendikson; Mcnaughton; Wilson *et al.*, 2017). Moreover, regarding MCP specifically, Monteiro, Nardi and Bastos

Filho (2012) highlight that the teaching of these topics in the teachers education must be reformulated, aiming to provide the future teachers the autonomy to address MCP topics in schools. In Brazil, despite the implementation of the BNCC, which brought new perspectives to the school curriculum, the general focus in the classroom is still on science developed until the 19th century (Oliveira; Rodrigues; Madureira; Silva *et al.*, 2020; Reis; Silva; Andrade-Neto, 2018).

In this sense, the present research becomes highly relevant. Firstly, it presents a series of resources and activities developed with high school students on the Relativity Theory. Moreover, it also investigates students' conceptions and understanding of the concepts worked on, analysing their mental representations, which are fundamental for understanding scientific concepts. Through the results obtained, it is possible to outline effective strategies for approaching the Relativity Theory in schools, fostering its inclusion in the classrooms.

3 LITERATURE REVIEW

Here is presented the literature review developed for this research.

3.1 SEARCH METHOD

The literature review was initially conducted in 2021 and updated in 2022 and 2023. The platforms used for the search were ERIC, Scopus, SciELO, and Google Scholar. On ERIC, only the term “general relativity” was employed. The search on Scopus was conducted using title, abstract, and/or keyword searches with the terms and operators “general relativity” AND (“teaching” OR “learning” OR “education”). The same terms and operators used on Scopus were applied on SciELO, with searches conducted in both English and Portuguese. For the search on Google Scholar, the same terms and operators were applied. As the most accessed and cited papers are presented in first pages of Google Scholar, the first 20 pages were delimited and analysed. Additionally, a search was conducted using the terms and operators in Portuguese.

The complete selection of articles was carried out through reading the titles and abstracts. Only papers published in journals or conference proceedings were considered. The review was updated in 2022 and 2023. The same search platforms, terms, and operators were used, restricting the period to 2022 and 2023, respectively. Regarding Google Scholar, only the first 10 pages were analysed, following the same criteria described before.

The objective of the review was to ascertain what is known from research studies on students’ conceptions of General Relativity Theory (GRT) and to identify resources that can be used in secondary schools to address it. During the reading process, many works were found to focus on didactically discussion of Relativity Theory concepts, without any experimentation, and some works focused on scientific communication. Therefore, these works were not considered. Furthermore, a portion of the works also presented proposals aimed at higher education, without investigating student learning or conceptions. As the scope of the present research concerns secondary education, these works were also not considered.

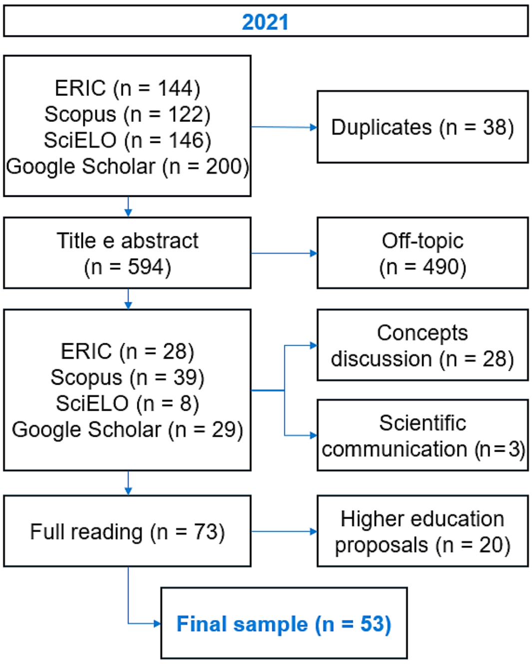
The inclusion and exclusion criteria are summarised in Table 3.1. Considering the established criteria, a total of 66 works were selected for the review, and these were read in their entirety. The screening process is presented in Figure 3.1 and Figure 3.2.

Table 3.1: Inclusion and exclusion criteria.

Inclusion criteria	Exclusion criteria
Journals or conference papers	Books, chapters, theses
Papers addressing Relativity Theory and teaching	Papers not in Portuguese, English or Spanish
Teaching proposal, teaching report and/or learning analysis	Discussion of RT concepts or scientific communication
Teaching proposal, and/or learning analysis in schools or learning analysis in higher education	Teaching proposal focused on higher education

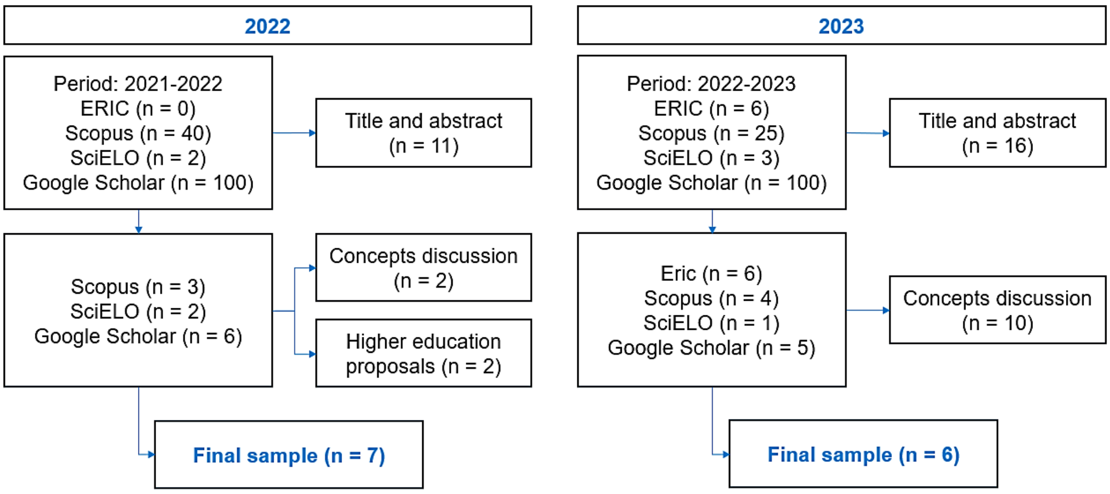
Source: Author (2024).

Figure 3.1: First selection process.



Source: Author (2024).

Figure 3.2: Second and third selection processes.



Source: Author (2024).

3.2 RESULTS

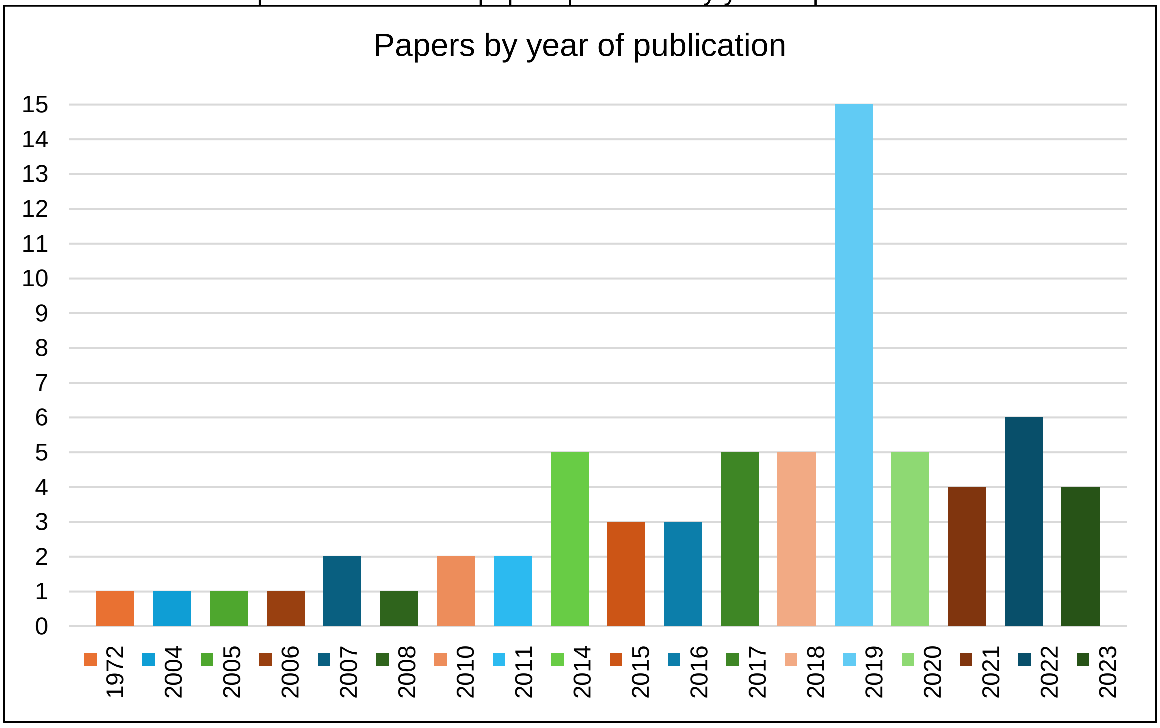
The results obtained from the 66 works are organised into three sections: trends in General Relativity teaching, teaching resources and approaches for General Relativity, and students' difficulties with General Relativity.

3.2.1 Trends in General Relativity Teaching

In this first section of the results, all the selected works from the review are presented in a general manner. The objective is to analyse the trends in General Relativity teaching in the selected studies, identifying the publication period, countries of origin, and approaches used.

Regarding the year of publication, the oldest selected article was published in 1972. Considering the number of published works, 2019 stands out with 15 works, followed by 2022 with six, less than half. The values can be observed in Graph 3.1. It is worth noting that, since the last update of the review was conducted in November 2023, the number of published works in 2023 may have increased after the date.

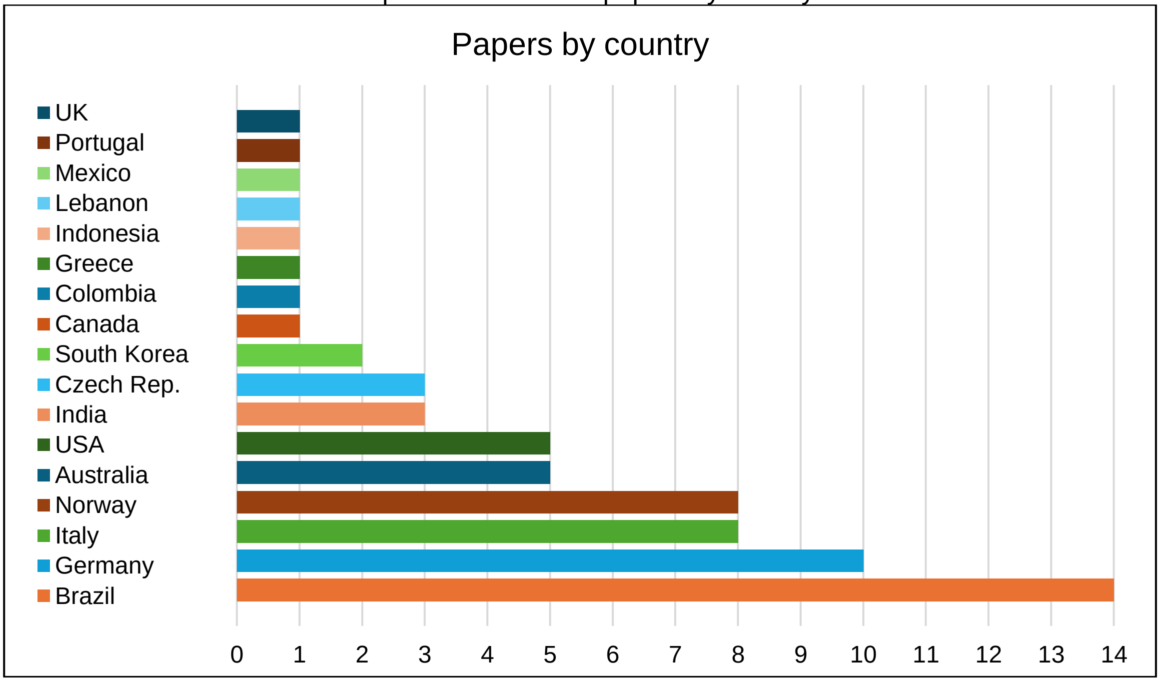
Graph 3.1: Number of papers published by year of publication.



Source: Author (2024).

Regarding the country of origin, a great variety was observed, with 17 different countries. Brazil presents the largest quantity, with 14 published works. Given that the review was conducted in Brazil, and databases such as SciELO were used, it might have facilitated access to these works. Other countries also present significant numbers of publications, with Germany having ten works, Italy and Norway having eight, and Australia and the United States having five. Graph 3.2 presents the distribution of the number of works by country.

Graph 3.2: Number of papers by country.



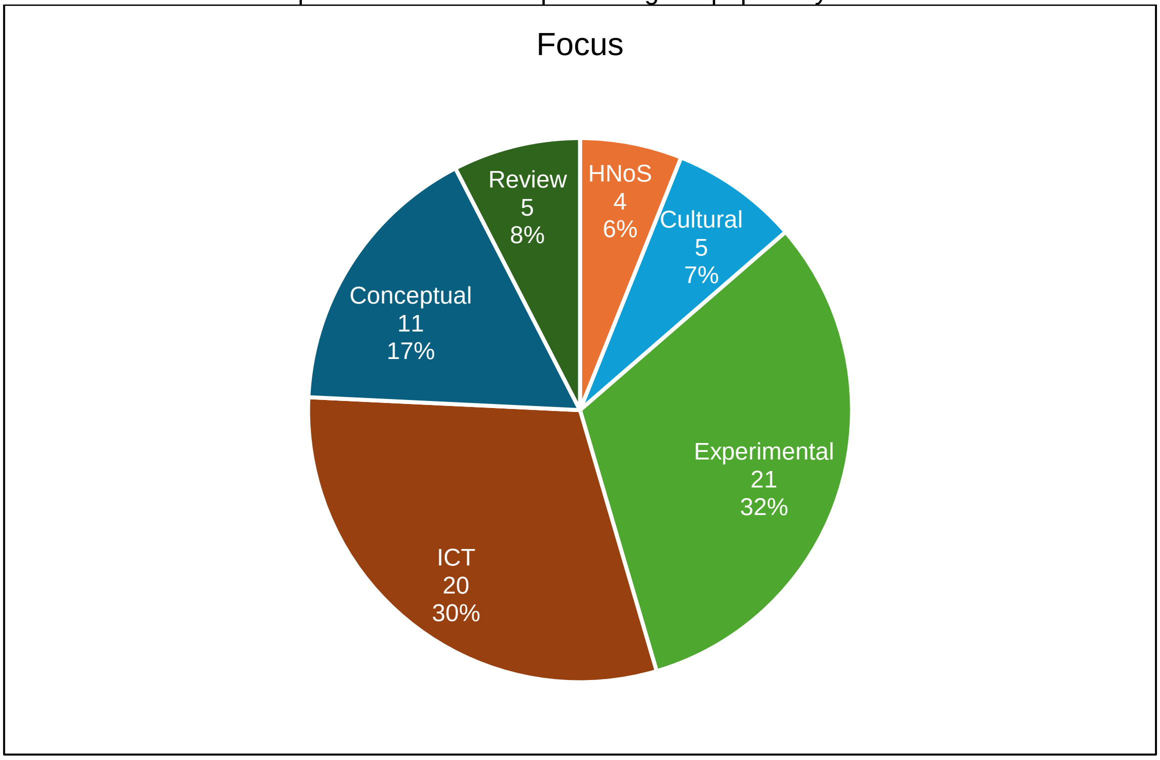
Source: Author (2024).

The aspect that provides the most information about the trends in General Relativity teaching is the focus of these selected works. To have an overview of what research has been done in the area and what gaps remain open, it is essential to consider the focus of the study in relation to two points: the main methodologies used for teaching approaches and the main aspects evaluated in students' conceptions.

After reading the selected works, it was possible to identify six different focuses in the studies: reviews, conceptual approach, use of information and communication technologies (ICT), experimental approach, cultural orientation, and history and nature of science (HNoS). All literature reviews, textbook reviews, and opinion surveys of teachers or students were included in the review category. All works that highlighted the concepts involved in General Relativity itself, using analogies and/or simplifications, were considered to have a conceptual focus. Also included in this category are works that did not report any type of intervention, only analysing the conceptions of a group of students.

Regarding the use of ICT, all works that used simulation software, applications, virtual reality (VR), or virtual learning environments (VLE) were included. All works that focused on the development of experiments or practical activities were considered to have an experimental approach. The cultural focus was present in works that used cultural elements to introduce General Relativity, such as paintings or films. Finally, HNoS encompasses works that introduce General Relativity through an approach to how science develops and/or analyse students' conceptions of the nature of science (regarding General Relativity). Graph 3.3 shows the distribution of the selected works in these focuses.

Graph 3.3: Number and percentage of papers by focus.



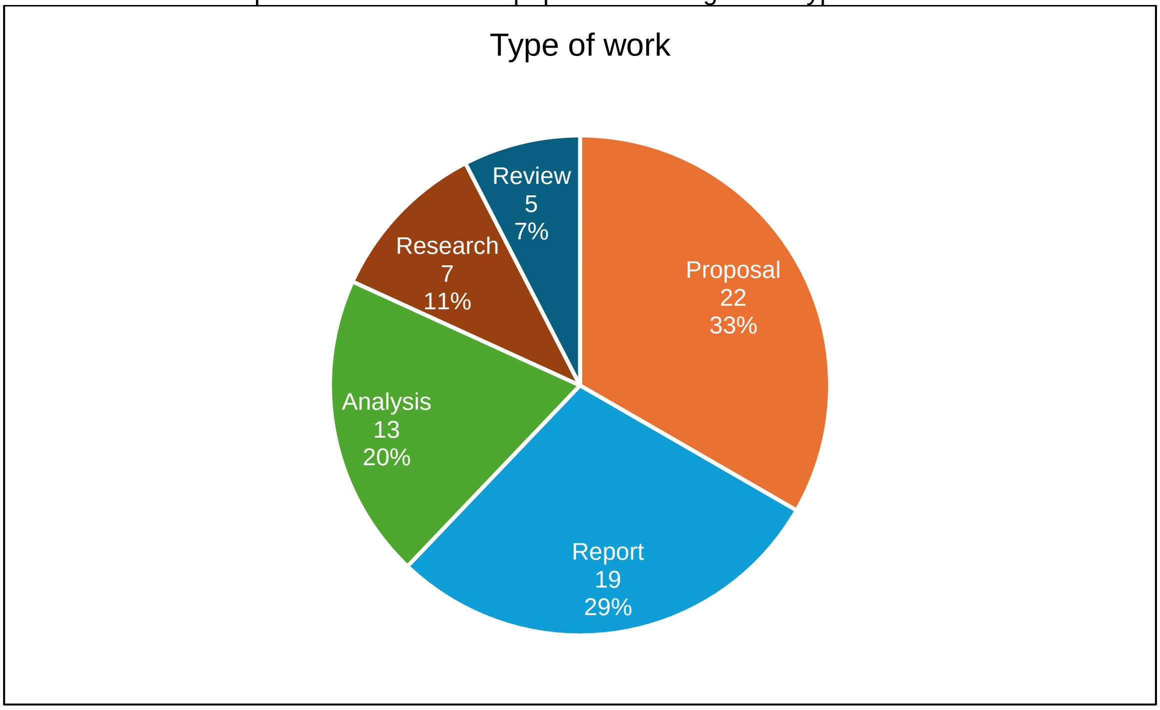
Source: Author (2024).

It is possible to note that most of the works are focused on the experimental approach (32%) and the use of ICT (30%). This demonstrates that there is certainly a wide range of practical activities and software available for use in General Relativity teaching. The HNoS focus presents the least expressive number, with only 4 works. Given that General Relativity is a complex topic, with which students and teachers experience difficulties, these numbers possibly reflect the focus for resources that can aid in understanding the phenomena addressed. Thus, more in-depth philosophical discussions are proposed less frequently.

Considering the type of work, it is also possible to observe some gaps. The works were classified according to the research activities that were carried out: proposal, report, analysis, research, or review. All works that present proposals for teaching activities, but without reporting any type of application, are classified in the proposal category.

Works that propose and report teaching activities, but without an in-depth analysis of the results obtained, are classified in the report category. On the other hand, works that conduct an analysis of conceptions, but without reporting any type of intervention, are classified in the analysis category. Finally, the most comprehensive works, which propose, report, and analyse the results, are classified in the research category. The review category includes the same works that were previously classified as such.

Graph 3.4: Distribution of papers according to the type of work.



Source: Author (2024).

In the distribution presented in Graph 3.4, it is possible to observe that most of the works focus on application proposals (33%), followed by application reports (29%). It is therefore perceived that there is a lack of works that conduct an in-depth analysis of the learning process of students and their conceptions about General Relativity Theory. In this sense, the importance of the research conducted in the present thesis stands out, as it brings a teaching proposal that was implemented and a detailed analysis of students' conceptions after the intervention.

3.2.2 Teaching resources and approaches for General Relativity

In this section of the results, the works which presented the most outstanding teaching resources or approaches are presented, which were considered to be more suitable for use in the research. A more comprehensive and detailed approach was elaborated as a review article (de Souza et al., submitted).

- Sector models

Zahn and Kraus (2014) presented what they call “sector models”, a resource to help visualize the curved spacetime according to General Relativity. The authors described the process of constructing the models and provided a link for free download. The models represent curved surfaces, first in two dimensions, using paper pieces, and later in three dimensions, with blocks, all through extrinsic visualization – from an external observer’s perspective.

Firstly, Zahn and Kraus (2014) demonstrated how two-dimensional sector models can be used to identify curvature as positive (spherical space) or negative (hyperbolic space). The authors compared sector models for a flat space, where there are no gaps between the pieces, and demonstrated how a spherical space will show “open” gaps, indicating positive curvature, and a hyperbolic space will show “closed” gaps, due to its negative curvature[1].

Next, the authors presented sector models representing a three-dimensional flat space (Euclidean) and curved spaces (around a black hole). Observing the three-dimensional blocks, it is possible to note how the pieces of the flat model fit perfectly, while the curved model shows some gaps, indicating positive or negative curvature[2]. Therefore, this model allows for extrinsic visualization of a three-dimensional curved space. The same type of model, as well as other variations, was used in subsequent works to represent geodesics and gravitational time dilation (Kraus; Zahn, 2019; Zahn; Kraus, 2019).

It is important to highlight the great difficulty faced in visualizing extrinsically surfaces with more than two dimensions. In everyday life, we experience three spatial dimensions with an intrinsic understanding of them. However, individuals face challenges in visualizing these three dimensions extrinsically. Regarding GRT, the

[1] See Figure 5.17 in section 5.2.3 of methodology.
[2] See Figure 5.18 in section 5.2.3 of methodology.

theory uses four dimensions for the universe, three in space and one in time. Even in an intrinsic view, humans are incapable of perceiving the fourth dimension.

- Paper cone

Highlighting the importance of using concrete demonstrations for addressing GRT, Ryston (2019a) introduce some activities for students to understand spacetime curvature. The activities presented by the author were used in workshops with high school students. The curvatures of time and space are initially addressed separately in a theoretical manner, then the activities for approaching space curvature are introduced.

The first activity is quite simple and consists of using a paper cone. A flattened paper cone is cut out, and a straight line is drawn on it, representing the flat space where objects move in a straight line in the absence of external forces acting upon them. When the cone is “closed”, the surface becomes curved, as well as the drawn line, and the greater the curvature of the cone, the more the line is curved[3]. This activity is quite simple and uses easily accessible resources, only paper, scissors, and a pen.

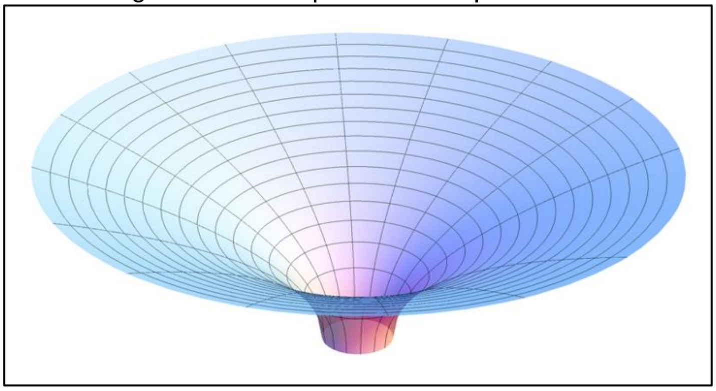
In the next activity, using the surface of a beach ball, it is possible to demonstrate how the line that connects the shortest distance between two points on it is the geodesic. Furthermore, objects moving on the surface of the ball will follow curved lines, so initially parallel lines can intersect. From a Newtonian perspective, which considers space flat, this occurs due to an attractive force. From an Einsteinian perspective, the lines simply follow the curved space.

A Flamm paraboloid (Figure 3.3), an embedding diagram, is used in the third activity. These diagrams consist of curved surfaces used to illustrate spatial curvature according to GRT. In the activity, a 3D-printed model of the paraboloid is presented to the students, who can interact with it. Subsequently, the students can also interact with a simulation presented in detail in another work by the author (Ryston, 2019b). These activities become a bit more restricted in terms of access to a 3D printer and computers.

Finally, using free-fall, the connections between space and time curvatures are presented using graphical representations with spacetime diagrams. Ryston (2019a) highlights that the workshop was well received by the students.

[3] See Figure 5.16 in section 5.2.3 of methodology.

Figure 3.3: Example of Flamm paraboloid.



Source: Peter (2022).

- Rubber-sheet analogy physical model

A well-known resource that aims to help visualizing relativistic phenomena is the rubber-sheet analogy. A model of this analogy was extensively used by the Einstein-First project, which focuses on teaching Einsteinian physics in schools. Within this project, Kaur, Blair, Moschilla, Stannard *et al*. (2017) described the use of the rubber-sheet model, made of Lycra, which they called a “spacetime simulator”, proposing various activities.

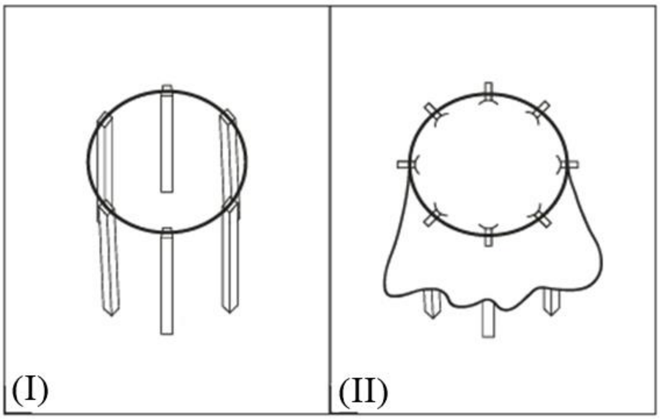
Firstly, the authors described a measurement of the distortion caused by masses in space. To do this, two points are marked on the Lycra sheet, and the distance between them is measured with a ruler. Students can measure the deformation and compare it to the increase in mass quantity. The second activity involves observing the movement of small balls around a central mass. Furthermore, using toy cars, an approach to gravitational lensing is described, where the car’s movement is curved, changing its direction, as if it was a light beam.

Kaur, Blair, Moschilla, Stannard *et al*. (2017) also proposed measuring the “gravitational” force on a test mass using a spring balance, a dynamometer. As the test mass approaches the central mass, the force increases, and it is possible to explore the Newtonian concept of gravity. The authors also described an activity to address geodesic precession, measuring the radius of the balls’ orbits, and gravitational waves from binary systems, using two large moving masses.

In the last activity presented, triangles are marked using magnets on a curved metal surface to discuss non-Euclidean geometry. Inspired by the Einstein-First project, Ruggiero, Mattiello and Leone (2021) also used the rubber-sheet model and triangles on curved surfaces with elementary school students. Instead of a metal surface with magnets, they used balloons of different sizes and tapes to mark the triangles.

Postiglione and De Angelis (2021a; b) also proposed the use of the rubber-sheet model, describing step-by-step the process of constructing the model (Figure 3.4). A 1.8-meter diameter Lycra structure was built, tested, and approved by a group of teachers who agreed to use it in the classroom. An e-book and activity cards were also developed to accompany the model. In addition to the construction process, Postiglione and De Angelis (2021a) proposed activities for its use.

Figure 3.4: Representation of the rubber-sheet model structure without (I) and with (II) the Lycra sheet.



Source: Author (2023).

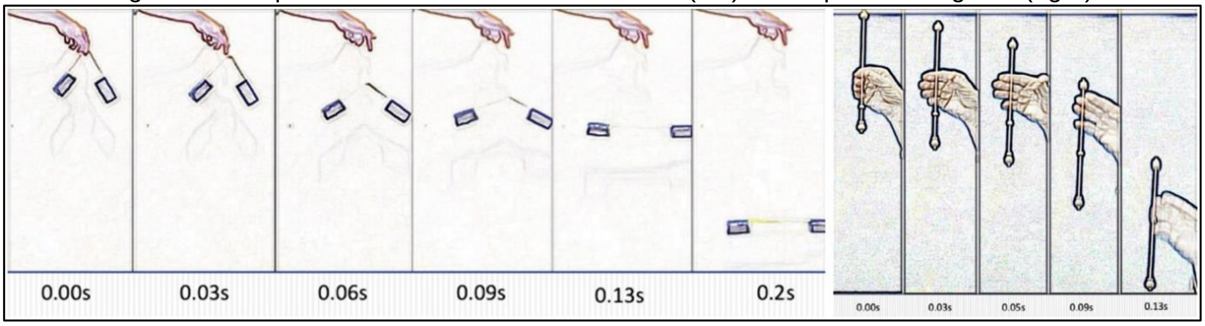
The above studies presented simple resources where the materials used can be easily adapted depending on what the teacher has available. However, the limitations of the analogy must be explicitly discussed with the students. It only allows visualization of deformation in two spatial dimensions. Moreover, it relies on Earth's gravity for demonstrations, which can lead to misconceptions. Therefore, it is crucial that, during the activities, the teacher engages in a discussion with the students about the theory and limitations of the analogy.

- Falling objects

In another work by the Einstein-First group, Boublil, Blair and Treagust (2023) reported some activities with falling objects to explain the Equivalence Principle. This principle states that, locally, it is impossible to distinguish a gravitational field from an accelerated reference frame. Therefore, an inertial reference frame is equivalent to free-fall.

In the first activity, the authors proposed observing a suspended spring being dropped. It is possible to observe that the bottom end of the spring only starts moving downward when the upper section has relaxed. For the second experiment, a cup filled with water with a small hole on the side was held. When the cup is dropped, the water stops flowing out of the hole because, in free-fall, the water inside the cup is experiencing zero weight. Boublil, Blair, and Treagust (2023) also presented an experiment using a flexible dumbbell (a metal strip with a piece of wood at each end) and repulsive magnets (toroidal magnets attached to a wooden rod) (Figure 3.5). In both experiments, the apparatus is dropped, and in free-fall, it acts as if there was no gravity.

Figure 3.5: Representations of the flexi dumbbell (left) and repulsive magnets (right).



Source: Adapted from Boublil; Blair e Treagust (2023).

These activities help students understand the fundamental concepts of Einsteinian gravity and free-fall. The abstract idea that free-fall is equivalent to the absence of a gravitational field involves changes of reference frame and can be observed using these external resources. Furthermore, with the different experiments presented, it is possible for teachers to use the most suitable ones for their context.

- Metallic globe

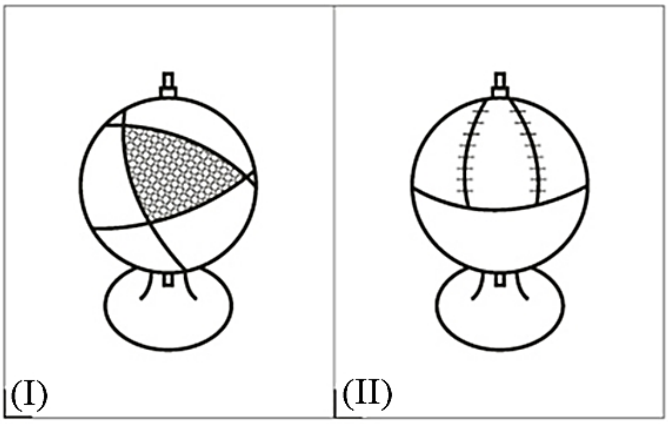
Andersen (2020) proposed ways to introduce General Relativity by addressing its geometry through three activities using a metallic globe (Figure 3.6). The first activity involves measuring the curvature of the globe using small magnets positioned to form a triangle. Using strings and a pencil, it is possible to calculate the curvature using geometry. It is essential to emphasize to students that they are calculating the curvature of a two-dimensional curved surface.

The second activity requires more complex mathematical tools and is slightly more challenging. Six triangles of decreasing sizes are drawn inside the marked triangle, and their hypotenuses can be measured. These values are compared to the values calculated using Euclidean geometry for flat and spherical shapes. By seeing how this geometry fails for a curved surface, students can perceive the need for a different geometry.

The connection between spacetime curvature and tidal acceleration is demonstrated with the third activity, based on a thought experiment from “Flatland” proposed by Thorne (1994), which introduces the geometric interpretation of gravity. If two objects are launched on a spherical surface from the equator towards a pole, they will meet at the pole. Andersen (2020) proposes demonstrating this situation using the globe. Two points are marked on the equator of the globe, and geodesic lines are drawn from them towards the pole; it is possible to observe that the two lines start to converge.

For a two-dimensional observer on the surface (intrinsic view), there is an attractive force between the objects moving on these lines that makes them converge. Therefore, it consists of a way to demonstrate, in a two-dimensional form, how gravity can be interpreted as a consequence of spacetime geometry rather than as a force. This activity is similar to the one proposed by Ryston (2019a) using a beach ball.

Figure 3.6: Representation of the metallic globe with the magnets (I) and geodesic lines (II).



Source: Author (2023).

The resources used are accessible, and even in situations where a metallic globe is not available, it can be substituted with a foam ball, for example. Since the universe is approximately flat in small regions, Euclidean geometry works well in everyday life, making it difficult to perceive the need for another geometry. By using the approach proposed by Andersen (2020), it is possible to demonstrate how the geometry that students are familiar with does not work for the universe that is curved on a large scale.

- Wine glass base

Regarding the concept of gravitational lensing, Huwe and Field (2015) presented a simple resource to allow visualization of these effects. The authors used a broken wine glass base with the centre covered with black tape. Using this tool and graph paper, several activities are proposed for understanding gravitational lensing through an optical lens model. However, it is worth highlighting that this is a visual metaphor to understand the concept, not an explanation of the gravitational lensing phenomenon.

- Group conceptual discussions

In a study that is part of the ReleQuant project, Steier and Kersting (2019) analysed the dialogue and gestures of two high school students while performing activities related to the rubber-sheet analogy. They highlighted the role of imagination in scientific education as a social process that helps students understand complex abstract topics through collaborative engagement, with words and gestures mediating the imaginative process. The authors emphasized that the main challenges for students are that the effects of General Relativity in everyday life are hardly perceptible and that it is necessary to learn to use three-dimensional representation systems. When students faced challenges, communication through dialogue and gestures helped them in the learning process.

In another work, also part of the ReleQuant project, Kersting and Steier (2018) described another investigation into student interactions. The authors developed a metaphorical and thematic analysis of the written responses of student groups while discussing the rubber-sheet analogy. The authors concluded that social interaction was crucial for students to signify abstract concepts. They also highlighted the importance of providing students with opportunities to engage in collaborative and creative activities that foster their understanding of curved spacetime.

- Science movies

The use of movie scenes not only helps students perceive and visualize phenomena but also captures their attention, resulting in greater engagement in the learning process. Such cultural artifacts can expand students’ cognitive abilities and facilitate their understanding of General Relativity Theory (GRT). From this perspective, Moura and Vianna (2019) and Almeida and Soltau (2022) used the movie *Interstellar* (Nolan, 2014) to address modern physics in high school education.

After diagnosing students' prior knowledge, Moura and Vianna (2019) used the movie alongside conceptual discussions to introduce GRT and modern physics topics from a new perspective. Notable scenes from the movie were the focus of discussions, such as the mission to Miller planet, which illustrates time dilation. In this scene, some characters approach a black hole, while others wait on a spaceship. When they return to the spaceship, although only a few hours have passed for them, more than 20 years have passed for those who waited.

In another perspective, Almeida and Soltau (2022) used the flipped classroom methodology with the film. Firstly, students watched *Interstellar* to motivate their participation in activities and promote initial questions. The following lesson focused on discussions, exercises, and experiments related to these materials. In the final activity, students created videos or posters about the discussed topics, explained them, and related them to the observed film scenes.

Students not only interacted with the movie but also with YouTube videos and texts and produced videos and posters. Movies are easily accessible and generally well-received by students. Consequently, using familiar cultural references is a valid resource that can stimulate their curiosity and make GRT more accessible.

- GR virtual environment

The ReleQuant virtual environment used by Kersting and Steier (2018) and Steier and Kersting (2019) is presented and explained in other works by the research group (Henriksen; Bungum; Angell; Tellefsen *et al.*, 2014; Kersting, 2019a; b; Kersting; Henriksen; Bøe; Angell, 2018; Kersting; Toellner; Blair; Burman, 2020). Within the environment, a freely accessible virtual model was developed.

The resource was built from a relativistic perspective of the Newton's first law, referred to as "Einstein's first law", where "objects that are not influenced by forces move along geodesic curves in spacetime" (Kersting, 2019a, p. 2). Free-fall is represented in a diagram where time curves, according to Einstein's interpretation of gravity[4]. The authors presented the functionalities of this model and how it can be used to complement the rubber-sheet analogy, where time is not represented, highlighting

[4] See Figure 5.10 in section 5.2.3 of methodology.

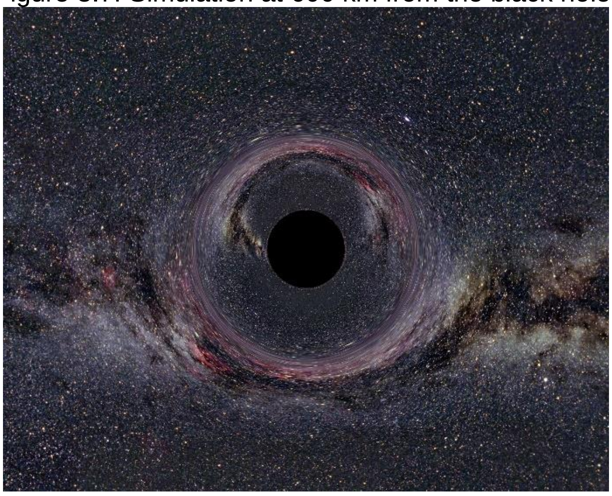
the importance of using different resources that complement each other to overcome learning gaps.

- Computational simulations

Again, using embedding diagrams, Ryston (2019b) presented possibilities for their use through a computational simulation, mentioned earlier. After introducing the mathematical equations that give rise to these diagrams, a visual Python software was developed. In the simulation, it is possible to observe the movement of small spheres on a surface curved by a massive object. Parameters such as the initial radius of the sphere and the central object's mass the can be changed, generating different results in the movements. The simulation is easy to manipulate and is available online[5].

In another work, Kraus (2008) presented first-person simulations for Special and General Relativity to facilitate the visualization of relativistic effects. Regarding GRT, the simulation consists of visualizing the surroundings of a black hole, where light deflection can be observed. In this simulation, a black hole was positioned in front of the Milky Way. The deflection of light emitted by the galaxy behind the black hole can be perceived as the observer approaches it (Figure 3.7). The simulation is free available online, and facilitates the visualization of the light deflection phenomenon, which is not observable in everyday life.

Figure 3.7: Simulation at 600 km from the black hole.



Source: Kraus (2008).

The interactive simulation *Universe Sandbox* was used by Ferreira, Couto, Filho, Paulucci *et al*. (2021) to demonstrate the collision of stars. First, between two

[5] See Figure 5.10 in section 5.2.3 of methodology.

stars with the same mass and with different masses; and then between five stars, one central massive star and four smaller stars at different distances. The authors also used the simulation to discuss the velocity of stars during the collision processes. *Universe Sandbox* is an interesting educational tool for visualizing gravity effects in different situations. However, a license must be purchased to use this software, which limits its use to only when payment is possible, which is not the situation of many public schools.

3.2.3 Students' conceptual difficulties with General Relativity

In this section, the main conceptual difficulties faced by students identified in the selected studies, both in basic education and higher education, are summarised. It is worth highlighting that, as pointed out in section 3.2.1, most of the identified works focus on proposals or application reports, without a more in-depth analysis of the results. Therefore, only the works classified as students' conceptual analysis and teaching research are considered in this section.

- Visualization

One of the main difficulties reported in the literature is related to the visualisation of relativistic phenomena. As observed by Steier and Kersting (2019), tensions between Einsteinian and Newtonian conceptions of gravity create difficulties for students in their imaginative processes. Therefore, these authors stress the importance of activities that stimulate collaborative imaginative processes to help overcome these conceptual obstacles. The difficulty in the abstraction process necessary for learning General Relativity was also reported by Ruggiero, Mattiello, and Leone (2021) with elementary school students.

Steier and Kersting (2019) highlighted another major challenge for students is the need to learn how to use three-dimensional representation systems. Since the concept of curved four-dimensional spacetime is essential for understanding General Relativity (Mcinerney; Sutton, 2024), this difficulty becomes an obstacle to learning.

A similar result was observed by Kim and Lee (2016) with pre-service teachers who presented limitations in spatial thinking. They observed difficulties among participants in understanding the four-dimensional curvature of General Relativity from two-dimensional images in textbooks. Still, regarding curved four-dimensional spacetime, Bandyopadhyay and Kumar (2010) observed difficulties among university students with the intrinsic view of an observer on a curved surface. Thus, the students tended to use an extrinsic view, i.e., a view from a higher dimension where the surface is contained, for example, using three dimensions to observe a two-dimensional curvature in a globe.

Kersting and Steier (2018), on the other hand, found that students relate their understanding of spacetime to their ability to visualise it, highlighting the importance of developing this ability. The lack of representation of time curvature in the rubber-sheet analogy, for example, can be compensated for by using simulations and animations to complement it. Moreover, these authors perceived that students tend to forget object movements in the temporal dimension. Even objects at rest are moving in time, and the time movement is necessary to explain free-fall, for example. Therefore, this limitation makes difficult to relate General Relativity ideas to everyday situations.

- Connecting GR to everyday situations

The difficulty of connecting General Relativity Theory (GRT) with everyday life was also observed in other studies by the ReleQuant group. Steier and Kersting (2019) highlighted that, despite being able to explain Einstein's theory, students view it as incompatible with their daily experiences. Since the effects of General Relativity on everyday life are hardly perceptible, contradictions arise between students' daily experiences and relativistic concepts, such as conceiving "gravity as a force" (Steier and Kersting, 2019).

It was also observed that, although students can recognise that both Newtonian and Einsteinian explanations for gravity are valid, they consider Einstein's view as too far from real life (Kersting; Henriksen; Bøe; Angell, 2018). This leads to difficulties in relating the geometric view of space and time used in the Einsteinian theory to daily experiences of gravity. The authors highlight that the concept of spacetime is engaging, but also challenging for students.

For example, Stannard, Blair, Zadnik and Kaur (2017) and Stannard (2018) pointed out that, although the Einsteinian idea of gravity is the most accepted, it is challenging to explain simple situations, such as free-fall, using it instead of Newtonian ideas. This is in line with Kersting's (2019a) proposal, which highlights the importance of addressing free-fall in both Newtonian and Einsteinian conceptions, and Kersting, Henriksen, Bøe and Angell (2018), who recommend that proposed activities explicitly connect phenomena with students' daily lives, such as the ones proposed by Boublil, Blair and Treagust (2023) mentioned in the previous section.

University students also struggle to apply GRT to explain phenomena, tending to use classical physics concepts to explain relativistic phenomena (Gousopoulos; Kapotis; Kalkanis, 2016). Moreover, the tensions observed by Steier and Kersting (2019) between classical and relativistic physics in high school students were also observed in university students (Bandyopadhyay; Kumar, 2010; 2011).

- Reference frames

The difficulties in transferring everyday situations to an Einsteinian interpretation also impact the understanding of the Equivalence Principle, fundamental in General Relativity. To comprehend the principle, it is necessary to analyse the phenomenon of free-fall from a new perspective, as well as understand the use of different reference systems to analyse the situation.

In this context, Kersting, Henriksen, Bøe and Angell (2018) observed that students are able to explain and connect the Equivalence Principle to relativistic phenomena. However, they struggle to accept that there is no experiment that can determine whether a reference frame is accelerated or under the influence of gravity, reflecting an idea of an absolute reference frame originating from classical physics.

Other alternative conceptions related to the Equivalence Principle were observed by Boublil, Blair and Treagust (2023) with Year 7 school students. Analysing the answers to some questions, it was observed that students believe that there is no mass in free-fall (or it cannot be detected), as well as the idea that gravity and acceleration are the same thing. With high school students, Machado e Nardi (2007) also identified difficulties in understanding the equivalence between a uniformly accelerated reference frame and a uniform gravitational field.

Students generally have difficulty switching reference frames when analysing phenomena, in addition to holding ideas of an absolute and privileged reference frame (Di Casola; Liberati; Sonegoc, 2015). These difficulties can become obstacles for students to understand free-fall experiments and the Equivalence Principle. Kersting, Henriksen, Bøe and Angell (2018) highlighted that there is a gap between knowing the definition of an inertial reference frame and being able to apply it to a situation.

The difficulty with inertial reference frames was also identified with university students (Bandyopadhyay; Kumar, 2011; Gousopoulos; Kapotis; Kalkanis, 2016).

Another difficulty observed by Gousopoulos, Kapotis and Kalkanis (2016) was a tendency for students to confuse Special Relativity and General Relativity, mainly regarding gravitational time dilation. Kersting, Henriksen, Bøe and Angell (2018) also observed that, although the concept was widely recalled by students, when explaining it, students tend to confuse classical and relativistic concepts, such as using observer relativity to explain time dilation.

- Time dilation

Moreover, high school students also struggle with visualizing time dilation and can only describe a movement in spacetime by separating the temporal part from the spatial part (Kersting, 2019a). Furthermore, despite being able to describe a geodesic, students have difficulty describing motion along it, confusing it with motion in spacetime. Based on these results, Kersting (2019a) highlights the need for the use of different resources that complement each other to overcome learning gaps.

- Gravity

Other misconceptions were observed by Postiglione and De Angelis (2021a) after using a physical model for the rubber-sheet analogy with students. Some ideas, such as the notion that only large amounts of mass will deform spacetime, and that gravity only acts “downwards”, persisted even after the activities (Postiglione; De Angelis, 2021b). Therefore, they stress the importance of discussing the limitations of this model with students.

Pitts, Venville, Blair and Zadnik (2014) also observed some alternative conceptions about gravity. Ideas about the absence of gravity on the Moon and in space, where objects would “float away”, were identified. Even at the university level, some students hold the conception that light has mass, being attracted by gravity and causing gravitational bending (Bandyopadhyay; Kumar, 2010).

This same idea that light can be “attracted” can be observed in the results presented by Ubben, Hartmann and Pusch (2022). The authors investigated the mental models of university students about black holes. Many students demonstrated imagining black holes as disks that “suck in” objects, and light, in space. Finally, Machado and Santos (2004) used a hypermedia resource with high school students. The authors identified difficulties in solving problems about gravitation that involve calculations using scientific notation.

3.3 LITERATURE REVIEW'S CONTRIBUITIONS TO THIS RESEARCH

The development of the literature review presented had three objectives: to obtain a general overview of research on the teaching of General Relativity, to identify teaching resources that could be used to develop this doctoral research, and to identify the main difficulties presented by students regarding the theory.

From the categorization of the selected works, it was possible to observe that most focused on proposal applications. A smaller number of works reporting classroom applications were found, and among them, a further reduced number of works provided analysis of results about students' conceptualisation obtained with the application.

It is worth highlighting that all works that deal only with the analysis of students' conceptions, without any teaching intervention, focus on higher education. This reflects the fact that General Relativity Theory is usually taught only at the higher education level. In this sense, when it comes to schools, an intervention is required to analyse the results obtained.

Regarding the identification of teaching resources, from the analysis of the selected works, it is perceived that there is a good availability of materials and resources accessible for use in the classroom. There are works presenting the use of simulations and software, analogies with experimental construction, films, and interactive activities among students. In this sense, the review proved fruitful for the development of activities and materials used in the research.

It was also possible to observe that the studies that analysed the learning of General Relativity by students converged in their results regarding the difficulties identified. Among the main obstacles in teaching General Relativity is the new perspective brought by the theory, which ends up conflicting with students' commonsense conceptions and their daily experiences. Students' prior knowledge often interferes and becomes a barrier to learning the new theory. Therefore, it is important that concepts are introduced gradually, and a relationship is established with students' experiences, to overcome these obstacles.

Moreover, various reports of students' difficulties with visualizing relativistic phenomena were found, since these are hardly perceptible in daily life. Therefore, the importance of using varied resources, through different forms of representation, is highlighted, aiming to facilitate the visualization of phenomena by students.

It is important that these more usual difficulties identified are considered during the planning of activities with students. On the other hand, many studies also pointed out that, in general, students present a great interest and disposition to learn about General Relativity concepts, which fosters engagement in activities.

4 THEORETICAL FRAMEWORK

This thesis was grounded in two main theoretical backgrounds that guided the research, data collection and analysis. These two backgrounds are presented here.

4.1 MENTAL REPRESENTATIONS

In the context of teaching relativity, it is essential to understand the different formats in which students can internalize and process information. Different formats facilitate the interpretation of different information. In the case of mental representations, they can be mainly divided into two formats: propositional and pictorial. Therefore, both forms of representation play crucial roles in student learning, helping them build a multifaceted understanding of complex and counterintuitive concepts, such as the Relativity Theory.

Given that the doctoral research focuses on investigating the mental representations constructed and used by students, it is essential to define the concept of mental representation adopted. In this sense, this chapter explores the relationship between internal and external representations, discusses some perspectives on mental representations, the conflict between these different views, and finally presents the perspective to be adopted. Additionally, some studies on mental representations in physics education are presented.

4.1.1 Internal and external representations

From lived experiences, individuals construct internal representations, or mental representations, to make sense of objects or phenomena. Therefore, a representation plays the role of a “substitute”, not as a copy of what is represented, but preserving and interpreting the considered important information (Strasser, 2010). In the classroom context, for example, pedagogical activities and educational resources consist of external representations of concepts to be learned. Interacting with these resources, internal representations are constructed in the process of students’ learning.

Any object can be used as a representation, both for other objects and for internal representations. Therefore, what characterizes a representation is not an intrinsic attribute of the object, but the fact that someone uses it to represent something. In this sense, a memory, for example, is a reconstructed representation of a previous mental representation.

It is essential to highlight that having a mental representation of something is not the same as having knowledge about it, as individuals must be able to operate such representations. The content of a representation can be true, precise, appropriate, consistent, or not. Therefore, a representation that fails to refer to the object or phenomenon represented can be understood as a misrepresentation, i.e., the established relationship between the representation and what is represented is inadequate.

Another important point regarding internal representations is that they cannot be directly accessed. In fact, an individual cannot observe the internal representations of another or even of themselves. However, these internal states affect human behaviour. Therefore, information about mental representations can be obtained indirectly through the external representations that individuals use to express themselves. This process of transforming an internal representation into an external one is generally referred to as “externalization”. However, Strasser (2010) emphasizes that the term does not reflect what really occurs, as it is the use of an external representation to refer to a mental representation. This process can occur through speech, writing, or drawing, for example, in situations where the individual seeks and is able to express their knowledge about something adequately.

Therefore, there are indicators that someone is capable of generating and using mental representations. These indicators point to when someone possesses some knowledge that, in turn, is represented in some way. However, the external representations used must be interpretable by others as well. Through tasks within a *Think Aloud* protocol (Van-Someren; Barnerd; Sandberg, 1994), for example, it is possible to discover which cognitive skills are involved in the process by evaluating the individual’s performance.

In this sense, internal representations can be used to explain how someone develops their understanding and their skills. These mental representations are theoretical constructs of cognitive science to explain the continuous processing of information in the human brain. Mental representations can be defined as forms of “imitating” the external world – from sensory experiences, people construct mental representations. They are functional descriptions of how the brain stores information.

Therefore, identifying mental representations (through “externalizations”) is highly useful for improving the quality of teaching and learning. Spector (2010) emphasizes that, considering that internal representations are fundamental for problem-solving, the development of forms to assess external representations of these mental structures is critical for supporting effective learning.

4.1.2 The nature of internal representations

The study of the human mind’s functioning is still an open field, where different perspectives are adopted to better explain individuals’ cognitive processes. Regarding internal representations, mental images, or pictorial representations, are a long-standing controversial topic.

Among the various perspectives on internal representations, two antagonistic views emerged in the 1970s: the propositionalist theory and the pictorialist theory. This clash of ideas grew in the 1950s and 1960s, when the first ideas of artificial intelligence (AI) emerged, seeking to explain human mental functioning through computer programming. The perspective emerged that, just like a computer, humans also process information, implementing theories as computational programs.

In this context, the view that mental processes have an exclusively symbolic-structural format, by propositions, was defended by Zenon Pylyshyn (2003). According to Pylyshyn, all information is processed and stored in the form of propositions, therefore mental images are reduced to interconnected logical structures. On the other hand, Stephen Kosslyn (1986) argues that mental images are a special type of internal representation and, therefore, deserve differentiated attention in their study. Kosslyn proposes that there are internal representations in a pictorial format, emphasizing that different formats of representations highlight different information.

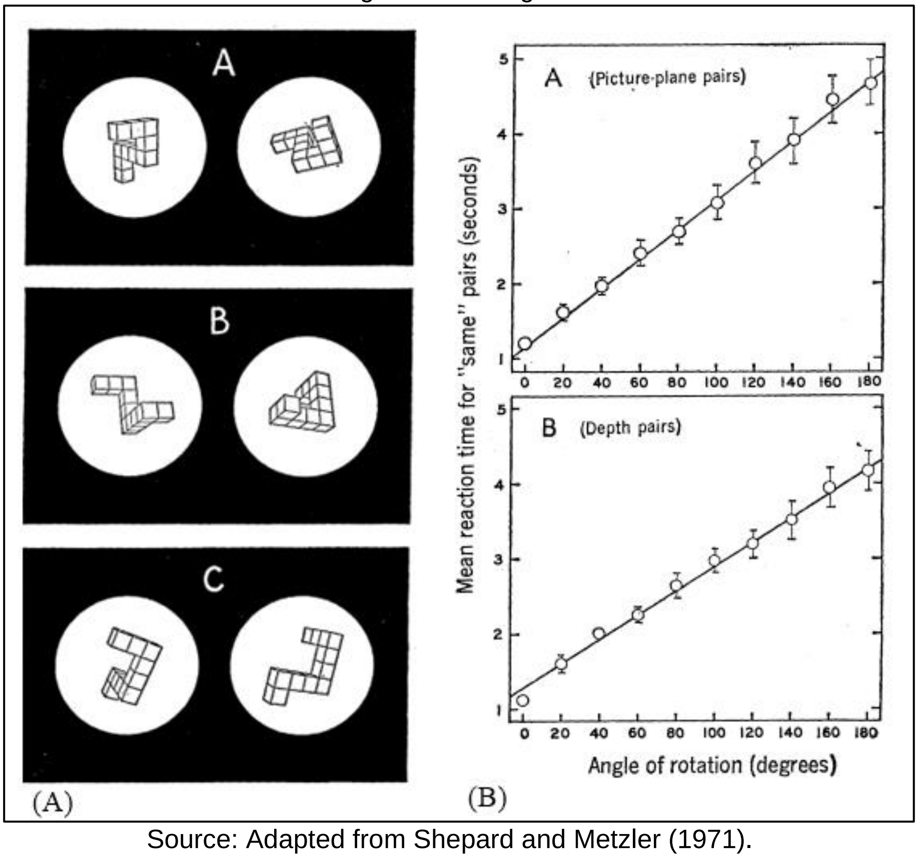
Although advances in neuroscience and brain imaging techniques have favoured the pictorialist view defended by Kosslyn, this philosophical debate about the nature of human thought continues.

4.1.2.1 The imagery debate

The discussion about internal pictorial representations, or the “image debate”, can be divided into four phases focused on different aspects of this discussion. The *first phase* begins with criticisms from Pylyshyn (1973) of the pictorial model, arguing that internal representations would be abstract mental structures, essentially conceptual, symbolic, and propositional. According to Pylyshyn, the "pictorial metaphor" would be mistaken in suggesting that mental images are entities to be perceived, whereas the representations used in thought would assimilate to language.

Some years earlier, the mental rotation experiment conducted by Shepard and Metzler (1971) presented results in favour of pictorial ideas. In this experiment, participants observed two-dimensional figures (Figure 4.1A) and had to evaluate whether the pairs were identical. It was observed that the longer the rotation angle of the object represented in the figure, the longer the time to complete the task (Figure 4.1B). According to Kosslyn and Pomerantz (1977), these conclusions validated the idea that the images seemed to rotate in people's minds, confirming the mental rotation hypothesis.

Figure 4.1: (A) figures used and (B) graphs relating the reaction time of individuals to the rotation angle of the images.

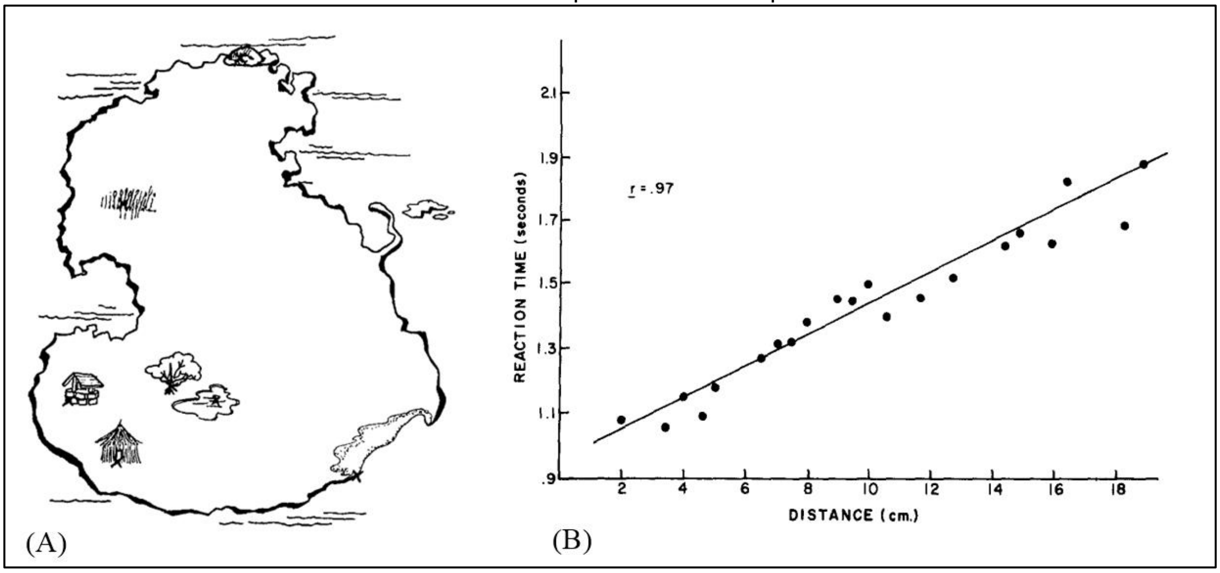


Source: Adapted from Shepard and Metzler (1971).

In his criticism, Pylyshyn questions whether mental images are primitive constructs or can be reduced to more basic structures in cognition. He emphasized that the fundamental structures of cognition are conceptual, symbolic, and propositional, rather than sensory or pictorial.

The *second phase* focused on discussions about empirical results and methodological problems, discussing how to interpret the obtained data in the best way. Pylyshyn's criticisms were responded to by Kosslyn with behavioural experiments, including the mental scanning of images (Kosslyn; Ball; Reiser, 1978). In these tests, individuals had to memorize an image of a map (Figure 4.2A) with certain points highlighted and report the time it took to "scan" from one point to another (Figure 4.2B).

Figure 4.2: (A) map memorized by individuals and (B) relationship between reaction time and distance between the points on the map.



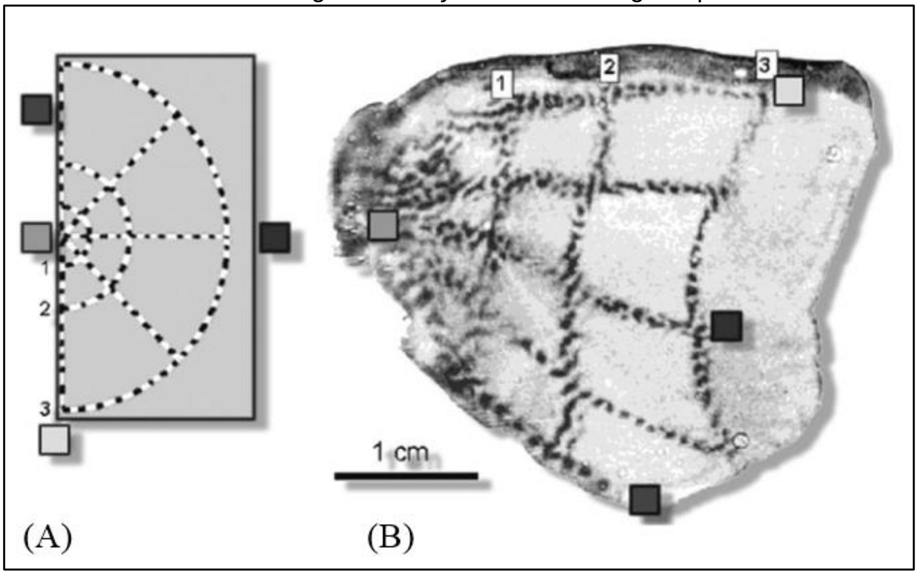
Source: Adapted from Kosslyn, Ball and Reiser (1978).

It was observed that the time to "scan" the map increased as the distance between the two points on the image increased. According to Kosslyn and collaborators, this result indicated that the mental representation had two-dimensional information (Denis; Kosslyn, 1999). However, Pylyshyn (1981) contradicted, arguing that the "tacit knowledge" of individuals would allow them to resolve such tasks. This would be an unconscious knowledge that individuals possessed about the memorized map, which would allow them to simulate what would occur through perception (vision).

The *third phase* brings neuroscience data to counter the criticisms of previous experiments. For example, in an experiment with a *Rhesus* monkey, it was possible to detect areas with higher cerebral activity (Figure 4.3B) while the animal observed a luminous pattern (Figure 4.3A)(Tootell; Silverman; Switkes; De Valois, 1982).

The obtained results demonstrated that the brain uses space in the cortex to represent the physical space of what is observed. Therefore, the neuroscientific data cannot be explained by "tacit knowledge" since they allow visualizing how information is actually processed in the brain.

Figure 4.3: (A) luminous pattern observed by the monkey and (B) monkey's cortex highlighting neurons with higher activity while observing the pattern.



Source: Adapted from Tootell, Silverman, Switkes and De Valois (1982).

Marking the beginning of the *fourth phase*, Pylyshyn (2002) argues that even with the neuroscientific data, there is no evidence of a "screen" for the mental image to be formed. Moreover, proponents of the propositional theory accuse proponents of the pictorialist theory of confusing the properties of the object being represented with the representation itself. Neuroimaging techniques allow observing areas with higher cerebral activity, but not the type of activity. For Pylyshyn, therefore, the information provided by these techniques about the activated cerebral location is not sufficient to clarify the relationships between cerebral activities and mental functions.

However, considering the neuroimaging results, Kosslyn, Thompson and Ganis (2006) argue that there are specific areas of the brain topographically organized that have the functionality of portraying patterns. These areas would be activated both by perception of the external world and during visualization of images. Therefore, these activated areas preserve the spatial structure of visual stimuli in the retina, even if distorted.

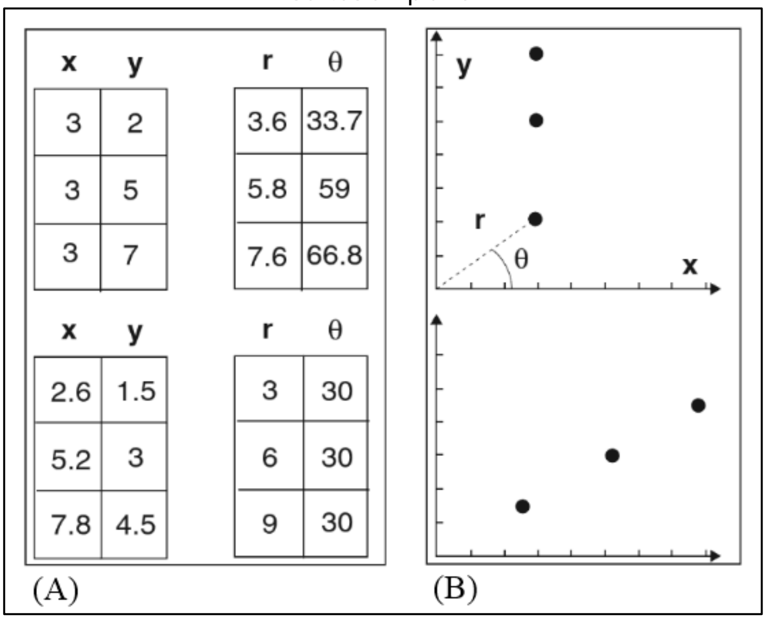
The discussions about whether mental images are a distinct type of internal representation are still open. For many, these experimental findings are not sufficient to accept the pictorialist theory. The topographic preservation of stimuli in cortex areas does not explain how the brain processes this information, i.e., how mental images originate. Therefore, researchers continue to investigate ways to identify the nature of mental functions.

4.1.2.2 Different Formats of Representations

Different forms of representation make accessible and explicit different information. Therefore, depending on the context, certain formats of representation will be more useful than others, considering the content to be presented. Kosslyn, Thompson and Ganis (2006) present a simple example using two formats of coordinates to represent points in space: Cartesian and polar (Figure 4.4).

The points (3, 2), (3, 5) and (3, 7) in Cartesian coordinates are the points (3.6, 33.7), (5.8, 59) and (7.6, 66.8) in polar coordinates. Observing the Cartesian coordinates, it is evident that the three points are aligned vertically. However, this information becomes difficult to perceive when using polar coordinates. On the other hand, the points (2.6, 1.5), (5.2, 3.0) and (7.8, 4.5) in Cartesian coordinates are the points (3, 30), (6, 30) and (9, 30) in polar coordinates. It becomes explicit that these points are on a diagonal using polar coordinates, information difficult to perceive with Cartesian coordinates.

Figure 4.4: (A) points in Cartesian and polar coordinates and (B) representation of points in the Cartesian plane.

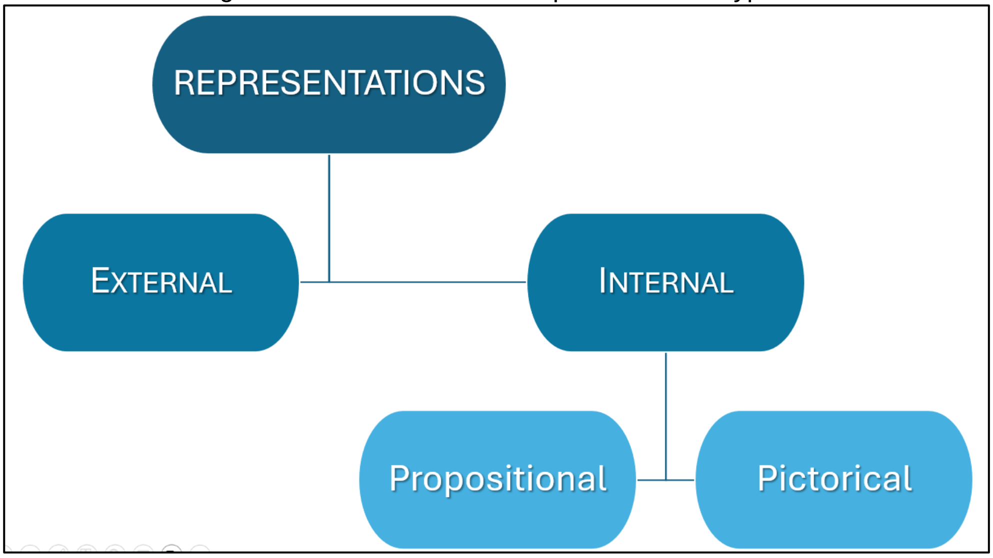


Source: Adapted from Kosslyn, Thompson and Ganis (2006).

Following the argument of this example, Kosslyn (1986) argues that mental representations can also be constructed in different formats, depending on which one is more useful for the information to be stored. Propositional representations are particularly effective for transmitting detailed and precise information, facilitating the clear expression of abstract concepts and the performance of logical operations, essential for solving theoretical problems and understanding complex causal relationships. In contrast, pictorial representations are valuable for visualizing spatial and temporal concepts, allowing students to perceive patterns, analogies, and transformations in an intuitive and holistic way. These representations become especially useful for understanding complex physical phenomena, such as relativistic effects, which are often counterintuitive.

Therefore, we adopt here a perspective aligned with the ideas of Kosslyn and collaborators, in which both formats of mental representations play roles in cognition. The Figure 4.5 presents a flowchart with the main types of representations that was considered for the thesis.

Figure 4.5: Flowchart of the representation's types.



Source: The author (2024).

- Propositional representations

Propositional representations, or non-pictorial representations, are conceptual and mediated by structures that assemble to language, although they do not depend on a specific language. These representations are also viewed as discrete or “digital” (Strasser, 2010).

The pictorial theory does not deny the existence of mental propositional representations. On the contrary, it affirms that pictorial representations can be accompanied by propositions that encode certain information, such as the colour of something represented, for example. In other words, verbal or abstract information can work together with imagistic information to generate mental images (Kosslyn, 1986).

> We stress that the depictive theory does not deny that propositional representations are sometimes used; instead, the claim is that depictions can also be used, in addition to propositional representations (Kosslyn; Thompson; Ganis, 2006, p. 29).

As highlighted earlier, different formats of representation can be more useful depending on the content to be transmitted or, in the human mind, stored. In this sense, propositional representations make explicit and accessible semantic interpretations, involving meanings. They are based on a symbolic format for coding knowledge, facilitating logical manipulation and precise communication of abstract concepts.

Therefore, propositional representations are abstract and can refer to entities and concepts that are not “imaginable”, such as feelings. According to Kosslyn, Thompson and Ganis (2006) “a propositional representations is defined to be unambiguous, unlike words or sentences in natural languages” (p. 11). Yet, these representations can also include aspects of perceptual qualities, such as the shape of an object.

However, for certain situations, the use of propositional representations may not be efficient. For example, describing a complex scene in terms of propositions and linguistic symbols would require enormous processing capacity. Depending on the task to be performed, different representational systems will exhibit differences in the nature, speed, and efficiency of the processing they support (Kosslyn; Thompson; Ganis, 2006). In this sense, for certain tasks, individuals may resort to using mental pictorial representations.

- Pictorial representations

Pictorial representations, or imagery, encompass mental images and simulations with visual schemas that allow for a more intuitive understanding of phenomena. They are considered continuous and analogical, linked to sensory experience (Strasser, 2010). Differently of propositions, they make explicit and accessible the perceptual aspects, such as colour and texture, of what is represented. According to Kosslyn, Thompson and Ganis (2006):

> Mental simulations are imagines scenarios that mimic what one would expect to happen in the corresponding actual situation, and depictive representations play a key role in such reasoning because they make *explicit and accessible* aspects of shape and spatial relations that otherwise need not be evident (p. 71, highlighted by the authors).

In cognitive science, internal representations are patterns of neural activation. Therefore, a mental image is a pattern of activation that generates the experience of "seeing" in the absence of a visual stimulus. It preserves the perceptual properties of that stimulus and is a subjective experience, meaning each one can interpret it in their own way. According to the pictorial theory, mental images and perception have similar properties, requiring a specific system for processing and interpreting pictorial representations.

Kosslyn (1994) asserts that "the properties of representations are necessarily defined in the context of a processing system, which includes both representations and processes that can interpret and manipulate them" (p. 4). Kosslyn's model is inspired by an analogy between the functioning of the human mind and how computers process information. According to the author, building a computer program that imitates the human mind is a good way to study it.

Therefore, just as a computer generates an image on a monitor from symbolic information stored, the human mind also generates mental images, but not necessarily on a screen. The mind uses a "functional space", a functional pictorial representation, where visual distances can be defined as information is processed (Kosslyn; Thompson; Ganis, 2006).

These images are processed by an internal observer, the "mind's eye", which performs the pictorial interpretation process. Areas of the cortex that are organized topographically are activated, not only during perception of the external world but also during visualization of images. Therefore, the "mind's eye" is a processor that interprets pictorial representations. When someone recalls a perceptual information, these interpretative processes are applied, and a mental image is experienced.

Images use space in the cerebral cortex to represent the space of the real world, that is, points are meaningfully organized topographically, representing corresponding locations in space. However, the patterns of cortical activation are not a "photograph" of what is observed. What matters is how the cortex interprets these distances, not their appearance to an external observer. Such as in the case of the Rhesus monkey experiment, although the pattern observed in the cortex seems deformed compared to the light pattern (Figure 4.3B), the cortex interprets it with the same shape of the light pattern. Therefore, the resolution power of the visual cortex limits the amount of information present in a mental image.

During the process of visual perception, stimuli within the individual's focus are compared to stored representations in memory. An object is recognized when the stimulus combines with one of the stored representations. Thus, when information is not encoded or stored in an individual's memory, it will not be possible for it to appear later in a mental image.

4.1.3 Mental representations in physics education

In physics education, it is recognized that identifying students' mental representations or models is crucial to understanding how they comprehend the concepts (Batlolona; Diantoro; Wartono; Leasa, 2020; Fratiwi; Samsudin; Ramalis; Saregar *et al.*, 2020; Özcan, 2013; Saglam-Arslan; Karal; Akbulut, 2020). By knowing students' mental representations, it is possible to identify alternative conceptions and plan better teaching strategies.

It is worth highlighting that a mental model is a specific type of mental representation that is more structured, allowing for the anticipation of events based on prior knowledge. The mental models are distinguished from other mental representations and support long-term and well-founded understanding of a concept or phenomenon (Al-Diban, 2012). Mental models usually focus on understanding complex systems, while mental representations encompass a broader range of cognitive structures used for various purposes.

Therefore, all mental models are mental representations, but not all mental representations can be considered mental models. Since this research investigates students' mental representations (in a broader sense), studies involving the investigation of mental models are also addressed here.

In general, when studying mental representations, categorizations are used based on the characteristics of these representations. For example, Ubben and Heusler (2021) and Ubben, Hartmann and Pusch (2022) classified students' mental models of science and black holes, respectively, according to "Gestalt fidelity" and "functional fidelity", proposing four archetypes. For example, regarding black holes, they identified models of the undeveloped type, where students have no idea how to imagine them, the architectural model type, which only considers appearance ("they are black"), the functional type, which prioritizes what they do ("they suck in matter"), and the dual type.

Investigating mental models of spin in quantum mechanics, Özcan (2013) used an epistemological perspective, classifying them as classical, quantum, or without a model. In another study involving concepts of force and velocity (Özcan; Bezen, 2016), the models were classified as Newtonian, Aristotelian, and impulsive. Fratiwi, Samsudin, Ramalis, Saregar *et al*. (2020) and Saglam-Arslan, Karal and Akbulut (2020) used similar categories in studies involving Newton's laws and the concept of work, respectively. They basically identified what they called 'scientific models', 'scientific synthesis' (with elements of alternative conceptions), and 'initial models' (dominant alternative conceptions).

Some studies also analysed the format of mental representations, whether propositional or pictorial. Trevisan and Serrano (2018), for example, investigated mental representations of wave-particle duality in Quantum Mechanics. It was observed that the classical perspective predominated among students, and divergences between propositional representations (with a wave-like tendency) and analogical representations (with a corpuscular tendency) emerged in explaining phenomena.

Regarding the resolution of kinematics problems, Ibrahim and Rebello (2013) identified the use of mainly propositional mental representations by students. Fazio, Battaglia and Di Paola (2013) found similar results, with the predominance of proposition use by students in explaining thermally activated phenomena. Other studies also investigate mental representations of thermal expansion (Vidak; Odžak; Mešić, 2019), the concept of light (Anjos; Serrano, 2024) and forces (White, 2012). Regarding the concept of heat, some studies analyse mental representations together with students' ability to express themselves using multiple representations (Prahani; Deta; Lestari; Yantidewi *et al.*, 2021).

Observing the aforementioned studies, it is clear that investigating students' mental representations by categorizing them is essential. Therefore, for the analysis of the data obtained in this research, they were classified into categories according to the consistency of the mental representations, identifying their format, whether pictorial or propositional.

4.2 COGNITIVE MEDIATION NETWORKS THEORY

Regarding the internal and external representations discussed in the previous section, the Cognitive Mediation Networks Theory (CMNT) offers a perspective that complements and enriches our understanding of mental representations. While studies on mental representations primarily focus on internal brain processes, CMNT considers how these representations are formed, modified, and used through interaction with the external environment.

According to this theory, individuals' cognitive structures are not solely the result of isolated brain processes, but rather emerge from a complex network of interactions between the individual and their sociocultural and technological surroundings. Therefore, this section explores how CMNT articulates with mental representations, situating them within the broader context of cognitive mediation.

This discussion aims to examine how tools, symbols, languages, and technologies not only influence individuals' mental representations but also become an integral part of their cognitive processes. This perspective enables a better understanding of how changes in the technological and cultural environment, particularly in the digital era, provide opportunities to fundamentally transform the way students think and process information.

4.2.1 The digital era

The Cognitive Mediation Networks Theory (CMNT) posits that the integration of Information and Communication Technologies (ICT) into society has caused significant changes in individuals' cognitive structures Souza (2004). The widespread daily use of electronic devices, such as smartphones, laptops, and tablets, by people of all ages and social classes is a notable phenomenon (Kepios, 2023). With constant connectivity, individuals can access an enormous amount of information through the internet instantly.

In the CMNT perspective, cognition is a phenomenon of information processing that occurs partly outside the brain. With the vast potentialities and functionalities brought by ICT and electronic devices, the way people manage their lives has changed, and consequently, the way information is accessed and processed has also changed (Papadakis; Collins, 2001).

According to Souza (2004), individuals need to develop new competencies and skills to manage the immense amount of information they receive daily. The author argues that, as individuals interact with hardware and software, cognitive growth occurs. Thus, in the Digital Era, new forms of thinking have emerged through access to hyperculture, as Souza (2004) notes:

> [...] we witness the emergence of a Hyperculture, where external mediation mechanisms include computational devices and their cultural impacts, while internal mechanisms include the necessary competencies for the effective use of such external mechanisms (Souza, 2004, p. 85, our translation).

This hyperculture constitutes a set of factors that is substantially different from what is traditionally viewed as “culture” and emerges from the Digital Revolution and its implications. The author suggests that the interaction with the environment, or mediation, shapes individuals’ thought processes, resulting in cultural changes. ICT are new external mediation mechanisms (the technology itself and its cultural impacts), with superior reach due to their specific potentialities. As new forms of external mediation emerge, individuals must develop the necessary competencies to use these resources and process information. According to CMNT, these competencies are the internal mechanisms, called *drivers*, that enable the creation and/or modification of mental representations.

Thus, the thinking associated with ICT use presents “logics and forms of representation [that are] analogous to these technologies” (Souza, 2004, p. 85, our translation). This “hypercultural thinking” is characterized by a strong mathematical-scientific logic, visual representations, elaborate classification and ordering systems, effective strategies for selecting essential information, and algorithms that process large amounts of information. Therefore, hyperculture is directly linked to the emergence of new forms of thinking, with cognitive gains.

4.2.2 The basis of CMNT

The Cognitive Mediation Networks Theory (CMNT) is defined by Souza, Da Silva, Da Silva, Roazzi *et al*. (2012) as a theory that is “a contextualist, constructivist, information processing approach to human intelligence that aims to provide a broad approach to cognition by […] [developing] a coherent theoretical synthesis of psychological theories and frameworks that are usually seen as separate” (p. 2321).

Thus, CMNT is founded on and brings a unifying synthesis of the theories of Jean Piaget (1960), Gérard Vergnaud (1997), Lev Semenovich Vygotsky (1989), and Robert Sternberg (1984) in a coherent manner. The theory expands on Piaget's concepts of assimilation and accommodation by including information processing provided by external structures outside the individual, such as physical objects or even another individual. Like the ideas proposed by Vergnaud, CMNT recognizes the importance of schemes and concepts for organizing knowledge, considering that internal schemes will allow for the effective use of external mechanisms for information processing.

In accordance with Vygotsky, CMNT emphasizes the social and cultural context in cognitive development. However, CMNT expands this view to include digital technologies and considers that these external agents are active components of cognition, not just influences on it. Finally, the theory views cognition not only as an internal adaptive process but as a system that includes external resources. Therefore, CMNT recognizes the importance of context, as in Sternberg's triarchic theory, with different types of intelligence, but considers external mechanisms as an integral part of intelligence.

People construct their knowledge through information processing in their brains. Therefore, when performing a mental task, humans incorporate mechanisms such as data storage and manipulation, which occupy "space" in their memory. Thus, for certain tasks, the human brain does not have sufficient processing capacity to handle the phenomena and situations that an individual experiences (Souza; Da Silva; Da Silva; Roazzi *et al*., 2012).

Humans can overcome these limits by expanding their cognitive capacity through "some form of extracerebral information processing" (Souza, 2004, p. 58, our translation), i.e., information processing outside the brain. By using external processing tools, such as written notes or electronic reminders, individuals can "free up" memory for task execution, increasing their brain's cognitive capacity (Souza; Da Silva; Da Silva; Roazzi *et al*., 2012).

This process occurs in various everyday situations, such as using reminders, which can be written on paper or even digital notes on a smartphone. Daily, humans use these external agents to achieve cognitive improvement. This ability to manage and apply knowledge to their advantage allows humans to evolve in their capacity to learn.

These organized external systems, i.e., structures in the environment with potential to be used as information processing mechanisms to complement human cognitive abilities, provide crucial assistance in performing complex tasks, such as understanding abstract concepts, like curved spacetime in General Relativity, for example. In these cases, individuals need to interact with external structures to construct knowledge.

Therefore, CMNT is also consistent with several theories commonly used in science education. In the same perspective of Cognitive Load Theory (Sweller, 2011), CMNT acknowledges the limitations of working memory. However, the CMNT focus is on how the brain manages this cognitive overload by using environmental structures as auxiliary computational devices. While Social Semiotics (Halliday; Hasan, 1985) emphasizes the use of different modes of communication for meaning-making, CMNT expands this idea, positing that external representations become part of an individual's cognitive "toolkit" through the internal development of *drivers*. CMNT is also aligned with embodied cognition perspectives by emphasizing the role of the body in cognitive processes (Amin; Jeppsson; Haglund, 2015), particularly through the psychophysical mediation.

Hence, CMNT introduces *Mediation* and *Extracerebral Information Processing* as resources that will aid cognition. *Mediation* refers to the process by which humans rely on external structures to complement the information processing of their brains. *External Information Processing*, on the other hand, is a form of distributed cognition where part of the cognitive workload is offloaded onto external structures. In this sense, the author presents concepts – from their own theoretical framework – such as "external mediation mechanisms" and "internal mediation mechanisms" to differentiate what would be external cognition outside the brain.

4.2.3 Mechanisms and Forms of Mediations

As mentioned above, *mediation* refers to the process of using external structures to aid the information processing of the brain. This process involves the external mechanisms, all the external tools that can be used to process information, and internal mechanisms, inside the brain.

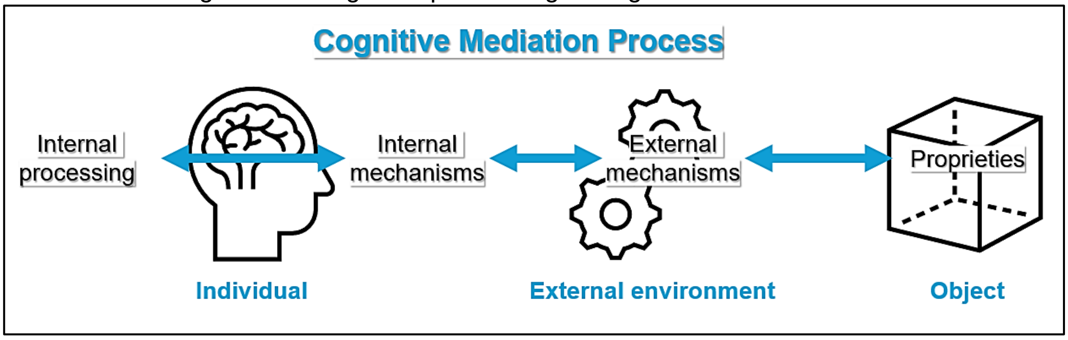
External mechanisms of mediation include all external resources available to the human brain that can be used to expand an individual's cognitive abilities. On the other hand, internal mechanisms needed to use these external resources are constructed through direct personal interaction with them. This process, by which an individual uses external structures to complete the processing of cognitive activities is defined as "mediation" (Souza, Da Silva, Da Silva, Roazzi *et al.*, 2012).

Therefore, there are several components involved in the mediation process:

- Object: something upon which the individual seeks to build knowledge (examples can be a physical item, a problem, or a relationship).
- Internal processing: the execution of basic logical operations by physiological brain activity.
- Internal mechanisms: mental structures of the individual that manage algorithms, codes, and data, allowing for the connection and integration of internal and extracerebral processing.
- External mechanisms: from simple physical objects to complex social activities, symbolic systems, and tools, which aid in information processing.

Figure 4.6 illustrates how cognitive processing occurs when interacting with external mechanisms to increase the capacity for information processing in an individual.

Figure 4.6: Cognitive processing through external mediation.



Source: Adapted from Souza (2004).

According to Souza (2004), external mechanisms "can only be effectively useful to an individual if they have a way of interacting with them effectively, according to the need and in an adequate manner" (p. 65, our translation). In other words, it is necessary for the internal structure of the individual to be able to translate inputs, outputs, and processing between them.

Therefore, when interacting with these external mechanisms, or resources, individuals develop new mental (internal) mechanisms that will allow them to understand the operations and information provided. This ultimately influences the way individuals think and reason, thereby acquiring new cognitive functions.

In CMNT, these internal mechanisms are called *drivers*, and they will enable access to information in new situations to solve different problems. Thus, the *drivers* will enable communication between the cognitive structure and the external mechanism of information processing. Through these interactions, the subject will understand the functioning of the mechanism to the point of internalizing the information contained within it.

Drivers can be seen as virtual machines that will contribute to the individual – in our case, the student – solving new problem situations (Souza; Roazzi, 2009). They *drivers* provide cognitive tools, techniques, and strategies for interaction with the external environment. External mediation, therefore, depends directly on these *drivers*:

> [...] cognitive mediation occurs if and only if there are internal mechanisms of support for mediation with the capacity for communication and control in relation to external mechanisms of extracerebral processing, that is, when the individual has, within themselves, a set of knowledge and skills that allow them to access and use such external mechanisms (Souza, 2004, p. 66, our translation).

In this way, external cognitive mediation is developed as a fundamental function for boost intelligence, connecting internal and external processes to the cognitive structure of individuals. The *drivers*, with their cognitive tools, techniques and strategies, will mediate the individual's use of the external mechanisms and the internal process within the cognition that enable the individual to use that external resource. Originally, the CMNT proposed four levels of external mediation: psychophysical, social, cultural, and hypercultural.

- Psychophysical mediation

Considered the most basic level of cognitive mediation, psychophysical mediation is conditioned by interaction with physical objects. Through interaction with objects in the environment, sensory-motor schemas are used, producing responses related to human instincts. This level of mediation occurs when external mechanisms allow for information processing through the relation of subject-object.

At this level of mediation, the physical components of the environment will provide a more efficient perception of situations. Therefore, the internal resources, *drivers*, required constitute basically the sensory-motor schemas of the individual. For example, when an individual separates the objects needed for the next day in the same location, he or she is using the environment to help them remember those objects. This constitutes an external psychophysical mediation.

In the classroom, the use of experimental activities or interactive models characterizes psychophysical mediation. Specifically, in the context of General Relativity, the physical model of the rubber-sheet analogy constitutes a psychophysical resource that promotes external mediation by helping students perceive the curved spacetime.

- Social mediation

Humans live in groups where characteristic patterns of behaviour and communication are developed. As individuals interact with each other, the perception of the environment ceases to be individual, being influenced by the perceptions of the other group members. In other words, individuals respond to the behaviour of others in certain situations.

Depending on the type of situation, an individual presents a certain posture, which is interpreted by others. Through this interaction between individuals, directly or indirectly, the social mediation level occurs. Therefore, this mediation can also occur through other mediations. In social interaction, the longer the existence of a group, the greater the tendency for improvement in the cognitive structures of the individual who is part of that group. This happens because social interaction within a group will contribute to the generation or modification of social *drivers*.

The durability of a group, therefore, makes the interactions between its members diverse. This, in turn, leads to the development of more complex forms of communication. As a result, language emerges in the form of speech and, subsequently, writing. Thus, it becomes possible to record information and events, writing documents, for example, leading to a more complex level of mediation, namely cultural.

In the classroom, when pair or group activities that promote collective engagement and interaction occur, social mediation is enabled. In these cases, the external resource used by a student to assist in information processing consists of a peer through discussions, for example.

- Cultural mediation

Through the use of language and its development with interpersonal interaction, the social mediation level is enabled. Whenever symbols for which meanings have been constructed are used, one has the use of external cultural resources. Therefore, this level of mediation encompasses the means of communication.

Complete ideas can be expressed through the use of emoticons in a conversation, or by traffic signs that assist drivers on a highway, for example. According to Souza (2004), “from the perspective of individual information processing, through culture, one has an extracerebral superstructure capable of performing operations of perception, memory, categorization, and learning” (p. 78, our translation).

In the school environment, whenever a textbook is used, for example, it enables cultural mediation. The use of language and images for communicating ideas makes the textbook a cultural external resource. Another example is the use of videos or films, which also communicate ideas using cultural elements.

It is worth noting that, even though both social and cultural mediation involve interaction fundamentally based on communication, often through language, they still having fundamental differences. Social mediation typically implies direct interactions between individuals or small groups, whereas cultural mediation, on the other hand, involves broader structures and practices that persist over time and across larger groups of people.

- Hypercultural mediation

As mentioned earlier, the digital era has led to the emergence of a hyperculture, causing significant changes society, and culture. Digital resources have enabled new external mechanisms of mediation, requiring *drivers* with new competencies for their use. According to Souza, Da Silva, Da Silva, Roazzi *et al*. (2012), this constitutes an additional stage in the cognitive evolution of society.

According to the theory, the emergence of hyperculture has brought new forms of interaction between social groups through digital technologies. There has been a change in the internal and external mechanisms used by individuals to process information, influencing their cognition. That is, to use a computer simulation, for example, the student must know how to operate and work with a computer.

Hypercultural mediation is made possible using digital technological tools that execute programmed and logical actions. These tools enable external information processing with distinct possibilities from other levels of mediation. Applications for daily task assistance, ranging from making simple electronic notes to the use of GPS for orientation, enable people to process external information, expanding the operations that they are cognitively able to perform.

In the classroom, regarding the teaching of sciences, the use of virtual laboratories or simulations through computers or even smartphones characterize a process of hypercultural mediation that assists students in processing information about specific topics being worked on.

General Relativity addresses various highly abstract topics that are difficult to perceive in everyday language and everyday situations. In this sense, the use of hypercultural external resources, such as animations or simulations, supports information processing, potentially assisting students in understanding the phenomena addressed.

4.2.4 Generative Artificial Intelligence and a New Mediation

Similar to hyperculture, the recent development and dissemination of Generative Artificial Intelligence tools have also brought profound changes in the way individuals process information and perform tasks. Although society is still adapting to the use of these tools, they have triggered genuine revolutions in the areas where they are implemented.

In this sense, the Cognitive Mediation Networks Theory has recently been updated to encompass these external resources. Thus, a new level of mediation has emerged, known as *Sophotechnic* mediation. This mediation is made possible when individuals interact with tools that possess the “capacity to process natural-language queries so as to translate them into commands […] producing desired content outputs” (Souza; Serrano; Roazzi, 2024, p. 4).

With these new external resources, which possess different potentialities than others, arises a demand for a completely new set of skills for an individual to interact efficiently with Generative Artificial Intelligence. Therefore, there is a need for the construction of new *drivers* that will influence the functioning of cognition in information processing.

4.2.5 The mediation forms evolution

According to the Cognitive Mediation Networks Theory (CMNT), external information processing happens when an individual interacts with one of the different types of mediation (Souza; Da Silva; Da Silva; Roazzi *et al.*, 2012). Considering the distinct forms of mediation presented previously, it is possible to observe they emerged in society successively. Therefore, these levels of mediation allow a glimpse into the society's cognitive evolution (Souza, 2004). The following Table 4.1 synthesizes this idea, showing the increasing complexity of the mediation mechanisms and the extracerebral processing involved.

Table 4.1: Evolution of CMNT mediation forms.

Mediation Level	**External Mechanisms**	**Internal Mechanisms**	**Extracerebral Processing**	**Teaching Relativity Example**
Psychophysical	Physical objects and environment	Sensory systems	Perception	Interaction with the rubber-sheet physical model changing the amount of mass or throwing balls and observing the movements
Social	Individuals' interaction	Social skills	Perception and memory	Group discussions about conceptual questions dealing with the Equivalence Principle, for example
Cultural	Symbolic systems and artifacts	Traditional and/or formal knowledge	Perception, memory, categorization and learning	Watching the *Interstellar* movie scene of the Miller planet where the gravitational time dilation can be observed
Hypercultural	Information technologies	IT domain concepts and skills	Perception, memory, categorization, learning, judgement, elaboration, and decision-making	Guided interaction with a computer simulation that shows relativistic effects such as the gravitational time dilation
Sophotechnic	Generative Artificial Intelligence tools	Prompts construction in natural language	Perception, memory, categorization learning, judgement, elaboration, decision-making, ideas creation and generation	Generation of the images illustrating relativistic concepts using an GenAI image-generator

Source: Adapted from Souza (2004).

In science education, particularly in teaching abstract concepts such as General Relativity, CMNT provides a useful framework for understanding how students develop their comprehension of complex scientific concepts. The external resources (e.g., textbooks, physical models, computer simulations) serve as computational aids that students internalize. Through constant interactions, students develop sophisticated internal mechanisms, the *drivers*, that enable their understanding and reasoning about abstract scientific phenomena.

Therefore, considering the premises of the Cognitive Mediation Networks Theory, the use of internal processing tools becomes crucial for learning, particularly considering the General Relativity, which is the central topic of this thesis. General Relativity deals with the idea of a four-dimensional curved spacetime inaccessible to students through their daily experiences. Moreover, much of the current scientific development in this field has been made possible using computers as external processing tools.

Thus, the activities to be developed with the students were planned to comprehend these levels of external mediation. It is expected that assisting students' cognition in processing information through these external resources can potentialize the construction of consistent mental representations by them in the learning process.

5 METHODOLOGY

Here all the methodological processes of this research are presented, encompassing from the planning of the research to the data analysis.

5.1 RESEARCH’S CONTEXT AND PARTICIPANTS

The research was conducted at the São João Batista State Technical School, located in the city of Montenegro, in the metropolitan region of Porto Alegre, Rio Grande do Sul (RS). The city of Montenegro was founded in 1873, and according to the Brazilian Institute of Geography and Statistics (IBGE)[6], the population is estimated to be 64,322 inhabitants (2022) with a Human Development Index (HDI) of 0.755 (2010). The city has experienced economic growth with the arrival of industries in the Southern Petrochemical Hub, located in the neighbouring city of Triunfo. Another sector responsible for the city’s economy is commerce.

The São João Batista School is an urban school located in the central region of the city (Figure 5.1), attended by students from all parts of the municipality, as well as some students from smaller neighbouring cities, such as Maratá and Brochier. According to the National Institute for Educational Studies and Research (INEP)[7], in the Socioeconomic Level Indicator (INSE), the school is situated at Level VI (2021), with Level VIII being the highest. The indicator is calculated based on the level of parental education, family income, and possession of goods. The INSE calculation uses data from the National Basic Education Assessment (ANEB), the National School Performance Assessment (ANRESC), and the National High School Exam (ENEM).

During the research period, the school had approximately 1,000 enrolled students, both in High School and Technical Education. The school offers technical courses in Chemistry and Electrotechnics, and therefore has specific laboratories for both. The school context and the daily life of the students were considered during the development of the research, being essential for its planning.

In addition to the researcher-teacher, the research participants were students from third-year High School classes (final year), aged between 16 and 19 years. The students were chosen because they were part of the classes where the researcher

[6] Data available in https://www.ibge.gov.br/cidades-e-estados/rs/montenegro.html (accessed on June 26th 2024).
[7] Data available in https://www.gov.br/inep/pt-br/acesso-a-informacao/dados-abertos/indicadores-educacionais/nivel-socioeconomico (accessed on June 26th 2024).

was the teacher in charge of Physics, and it was the academic year in which Modern Physics teaching was scheduled according to the school's teaching plan (Annex B), making this a convenience sample.

Figure 5.1: Front of the school.

Source: School's Facebook page (accessed on June 26th, 2024).

All students authorized their participation, or had authorization granted by their guardians in the case of minors, according to the Free and Informed Assent Form (Appendix K) and the Free and Informed Consent Term (Appendices L and M). This research was approved by the Human Research Ethics Committee of the Lutheran University of Brazil, via "Plataforma Brasil", under the number CAAE 52435921.0.0000.5349 and opinion 5.113.032.

5.2 TEACHING RESOURCES

As highlighted earlier, see chapter 2 section 2.4, it is common for topics in Modern and Contemporary Physics, including the Relativity Theory, not to be brought into the classroom in Basic Education. Therefore, addressing such topics in schools requires a transition and adaptation of scientific knowledge to be taught and learned, what can be referred to as Didactic Transposition (Chevallard, 1991).

According to Chevallard (2013), the knowledge taught in school undergoes an adaptation process for its teaching, since all knowledge originates from the scientific field. Chevallard (1991) has identified three levels of knowledge: "scholarly knowledge", what is produced by the scientists and researchers; "knowledge to be taught", the knowledge that is presented in teaching programs and teaching books; and "taught knowledge", representing what is effectively taught in the classroom.

The Didactic Transposition happens in two steps. Firstly the “external transposition”, that is the transformation of scientific knowledge (“scholarly knowledge”) into knowledge present in didactic materials (“knowledge to be taught”). Then the “internal transposition” is made by the transformation of the “knowledge to be taught” to the knowledge present in the school context (“taught knowledge”) (Chevallard, 1991).

For example, considering the Special Relativity Theory (SRT). The “scholarly knowledge” includes the original Einstein’s paper, “On the electrodynamics of moving bodies” with the two postulates and mathematical formulation (Einstein, 1905). This knowledge is adapted for teaching materials, focusing on the main consequences and simplified formulas, using analogies and highlighting its practical and technological applications, being transformed into “knowledge to be taught”. Finally, the teachers adapt what is presented in the materials to their students’ level of comprehension, select visual resources to be used and connect it to students’ previous knowledge, generating the “taught knowledge”.

It is essential to emphasize that the didactic transposition process is more than a mere simplification of knowledge, it is a process of constructing new knowledge corresponding to different epistemological domains (scientific field and classroom). Therefore, such adaptations are necessary to address how Relativity Theory is taught and learned in schools. Expanding the perspective of didactic transposition, Clément (2006) included the reference to social practices and values in the process.

Since teaching within the school must accompany scientific and societal development, the need for this Didactic Transposition process to occur with Modern and Contemporary Physics topics is highlighted:

> The new knowledge that emerges in scientific research and is used by industries and new technologies can be contained in textbooks, creating an approximation between academic production and what is presented in school. [...] The modernization of school knowledge is a necessity, as it legitimizes the discipline’s program, guaranteeing its place in the curriculum (Brockington; Pietrocola, 2016, p. 11, our translation).

In this sense, both the strategies to teach Relativity Theory and the materials used in this research were planned considering the precepts of the Theory of Didactic Transposition. Therefore, not all topics of the RT are addressed with the most exact and accepted explanation by the scientific community. It is a complex theory with which even university students have difficulties. Therefore, the focus of the developed activities is to provide students with a general and conceptual understanding of the theory's main aspects for this level of schooling.

To obtain relevant and reliable results, all application and data collection materials must be carefully elaborated. For this research, questionnaires were developed to be used as conceptual tests for data collection. Computational simulations and their written guides were also developed and selected. Experiments were developed, and a slide presentation to guide the classes was elaborated. All selected materials could be identified through the literature review conducted. All the materials produced and selected are described in the following section.

5.2.1 Conceptual Tests

The tests used aimed to analyse the students' conceptual understanding of the phenomena. The questions from the Special Relativity Theory (SRT) test used in the master's research were revised (De Souza, 2021). The original test had eight questions – two questions about spatial contraction, two about time dilation, and four about the confidence in each answer.

SRT space contraction – The situation in the first two questions presented two trucks on a highway. One was stationary relative to the road, while the other truck approached it at 80 km/h in the first question and at 0.7c in the second. The question asked about the distance measured by the moving truck in relation to the road, for each velocity. These questions allowed for the assessment of whether the student understood the phenomenon of spatial contraction, which is influenced by velocity.

SRT time dilation – The situation in the next two questions addressed a scenario similar to the twins' paradox. A friend was traveling by ship while the other waited for their return. The ship travelled first at 30 knots (56 km/h) and then at 0.7c. The question asked about the trip duration for the friend on the ship, for each velocity. These questions evaluated the student's understanding of the phenomenon of time dilation and the influence of velocity on it.

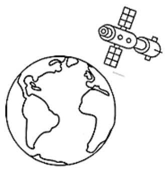
For the doctoral research, two more questions were added, addressing time dilation in a different situation. In the proposed situation, a person observed a ship passing by the coast while a person on the ship performed an activity. The question asked about the interval of that activity for the person on the ship. The inclusion of this new situation aimed to verify misconceptions related to the previous situation (of the friends) that might lead to a correct answer.

Furthermore, adjustments and corrections were made to the other questions, and the questions related to the students' confidence in their answers were removed, as they would not be included in the analysis. All SRT questions were multiple-choice. After revising the SRT questions, we started the development of additional questions related to the General Relativity Theory (GRT). Five questions were elaborated, four of which were multiple-choice, and one open-ended question that could be answered with drawings, diagrams, or text.

General Relativity Theory – The situations presented in the first two GRT questions addressed gravitational time dilation. Firstly, while an astronaut waited at the space station, the other goes to the Earth to perform an activity. In the next question, one astronaut waited in a spaceship while the other goes closer to a black hole. In both questions, the time interval for the astronaut who waited at the space station and for the one on the ship was asked. These questions aimed to evaluate the students' understanding of gravitational time dilation and its relation to the mass of the celestial body.

Figure 5.2: Test's questions about GR.

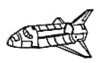
Q7. Two astronauts are in the internationa space station. Consider that one of therr down to the Earth, do an activity anc finished it in 8h, according to his watch What was the time interval for thé astronaut who stayed in the space station?



a) Also 8h.
b) Little more than 8h.
c) Much more than 8h.
d) Little less than 8h.
e) Much less than 8h.

Q8. Consider now that the astronauts are on a spaceship close to a supermassive black hole. One of them goes in a mission and gets closer to the black hole, finishing the mission in 3h, according to his watch. What was the time interval for the astronaut who stayed on the spaceship?

a) Also 3h.
b) Little more than 3h.
c) Much more than 3h.
d) Little less than 3h.
e) Much less than 3h.



Q9. Two points close to the Earth have between them a distance of 1km. What would be the distance between these points if they were close to a black hole in an altitude of 500km?

a) Also 1 km.
b) Little more than 1km.
c) Much more than 1km.
d) Little less than 1km.
e) Much less than 1km.

Q10. Two points close to the Earth have between them a distance of 1km. What would be the distance between these points if they were close to a black hole in an altitude of 20km?

a) Also 1 km.
b) Little more than 1km.
c) Much more than 1km.
d) Little less than 1km.
e) Much less than 1km.

Q11. Explain, as you were explaining to a classmate, why the Earth moves in an orbit around the Sun. To do that, you can use text, drawing, graphics, diagrams etc.

Source: Author (2024).

The next two questions addressed the same situation, where the distance between two points near a black hole was compared to the measurement made on Earth. Firstly, at 500 km from the black hole, and then at 10 km from it. The objective of these questions was to analyse students' understanding of the spatial deformation caused by a massive object and its relation to the distance from it.

The last question, an open-ended question, asked about the explanation for why the Earth moves in an orbit around the Sun. Students could explain their understanding of the phenomenon in the most convenient way for them. This question aimed to verify if students were able to understand gravity as the geometry of spacetime, according to GRT. All the questions comprised a single test with the final version at Appendix A. The questionnaires were applied as pre-tests before the activities began and as post-tests after their conclusion. The Figure 5.2 presents the questions of the test dealing with GR.

5.2.2 Simulations

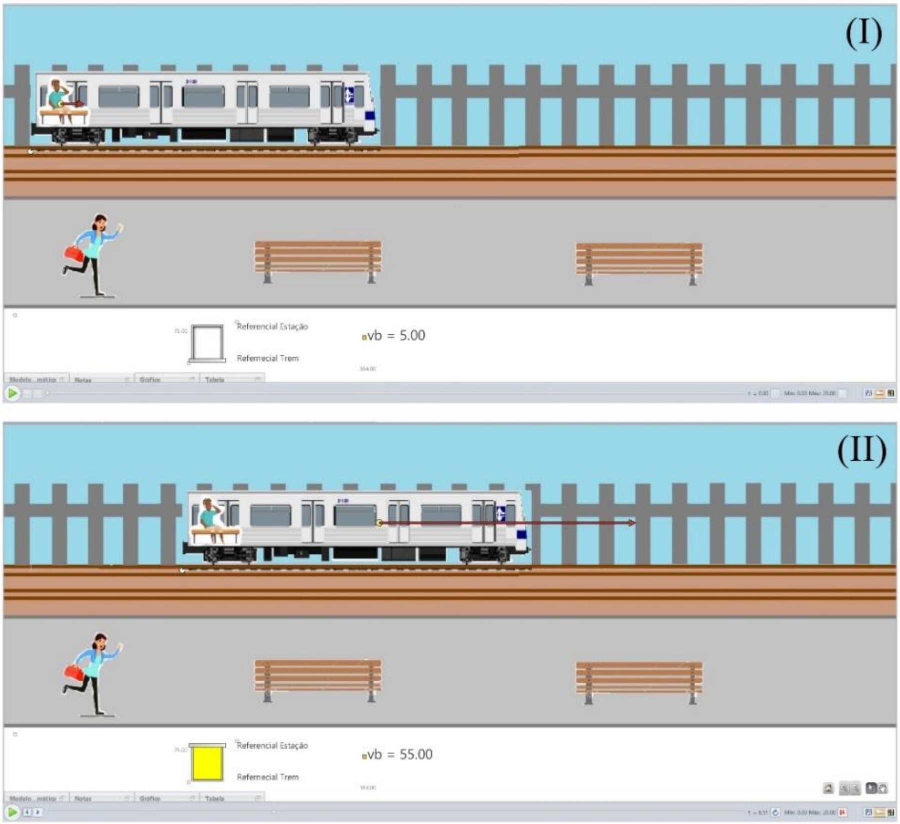
The four simulations developed in the master's research using the *Modellus 4.01* (Teodoro; Vieira; Vieira, 2004) software were revised and rebuilt using the more updated *Modellus X* software for "Cohort 2". These simulations are "Ball on the Train" and "Cars and Plane" to address Galilean relativity, and "Time Dilation" and "Space Contraction" for Special Relativity.

Simulation 1 – The "Ball on the Train" simulation (Figure 5.3) shows a boy throwing a ball forward inside a moving train. It is possible to change the observation reference frame, either the train's reference frame or the station's reference frame and observe the change in the ball's motion and velocity.

Simulation 2 – The "Cars and Plane" simulation (Figure 5.4) was developed based on the simulation used by Monaghan and Clement (1999) to observe the relative motion between two cars moving in opposite directions and a plane flying over them. It is possible to vary the plane's velocity using a controller and change the observation reference frame of the objects' motion, which can be the ground, the plane, car 1, or car 2.

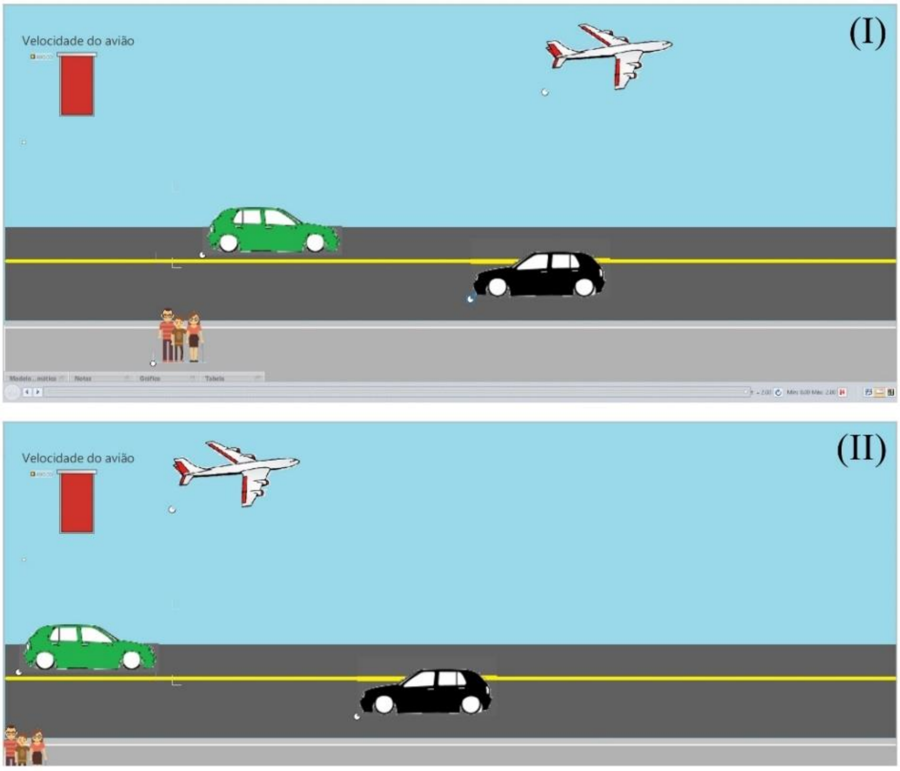
Simulations 3 and 4 – Both the "Time Dilation" (Figure 5.5) and "Space Contraction" (Figure 5.6) simulations presented the same situation of a train passing by a station, where the train's velocity relative to the ground could be altered. Depending on the velocity, it was possible to observe a clock on the train and one at the station (both observed by someone at the station) or the measured length of the train, for someone inside the train and someone at the station.

Figure 5.3: Simulation “Ball on the Train” in the train’s (I) and station’s (II) reference frames.



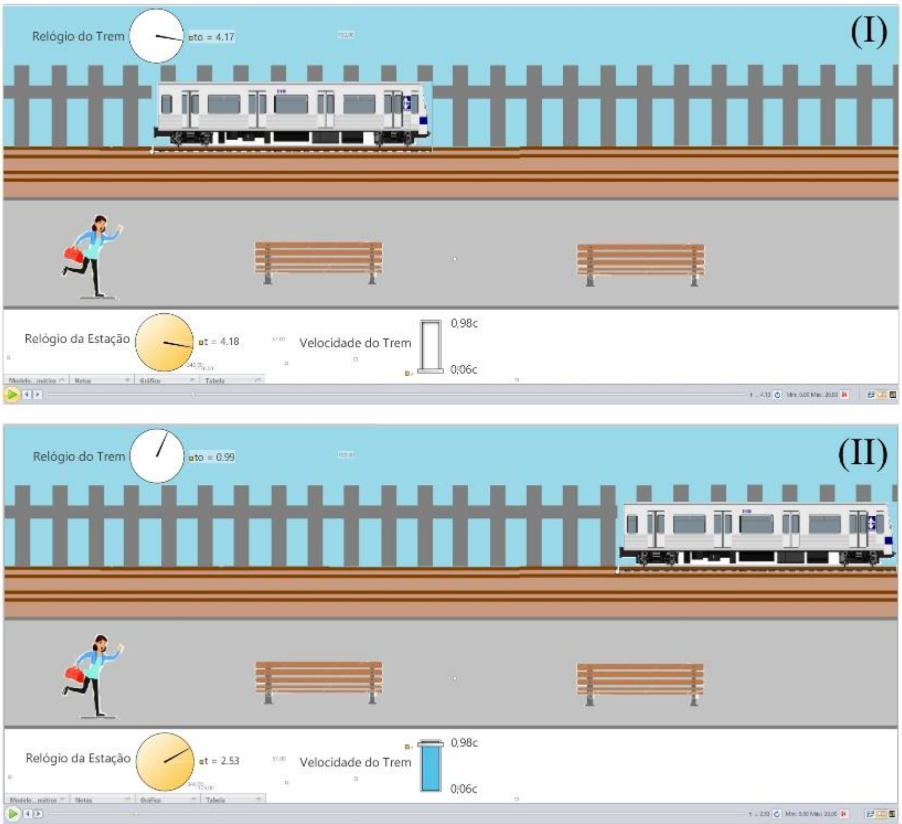
Source: Author (2024).

Figure 5.4: Simulation “Cars and plane” in the ground’s (I) and plane’s (II) reference frames.



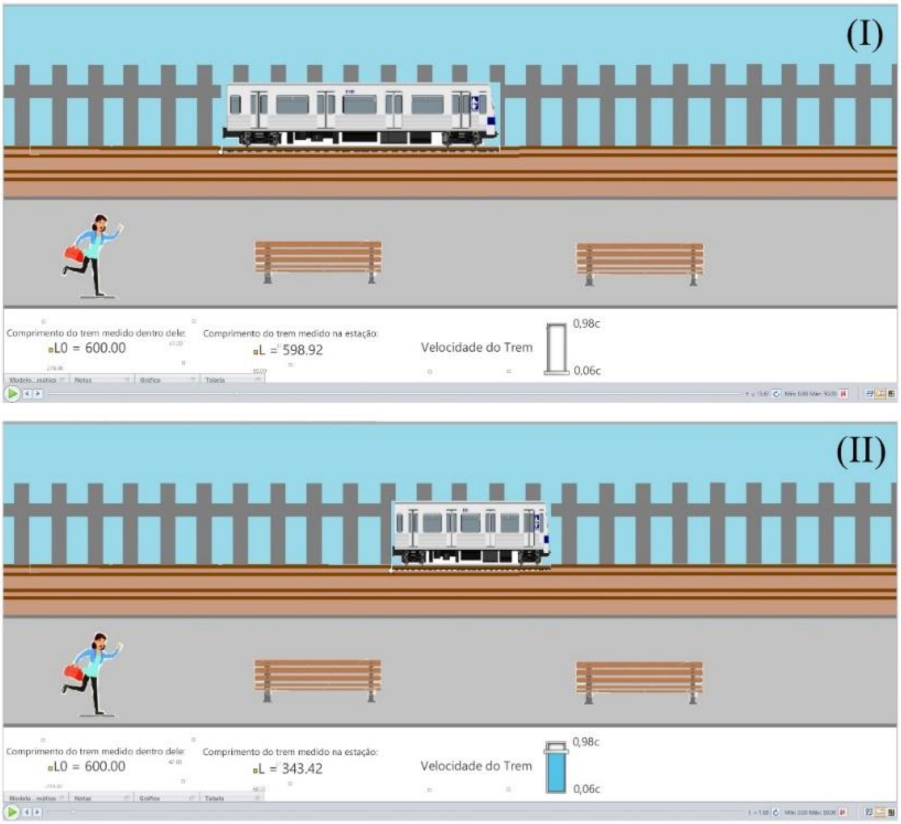
Source: Author (2024).

Figure 5.5: Simulation “Time Dilation” showing the train at 0.06c (I) and 0.98 (II) related to the ground.



Source: Author (2024).

Figure 5.6: Simulation “Space Contraction” showing the train at 0.06c (I) and 0.98 (II) related to the ground.

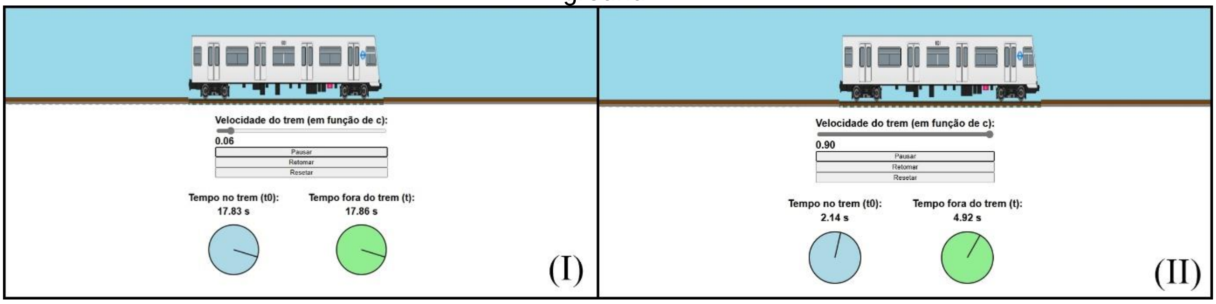


Source: Author (2024).

For observation purposes, the speed of light in a vacuum (c) was considered to have a value of 500 km/h. Originally, this information was visible to the students. However, with the revision of the simulations, the real value was hidden, and values were shown in terms of c in the simulation, ranging from 0.06c to 0.98c.

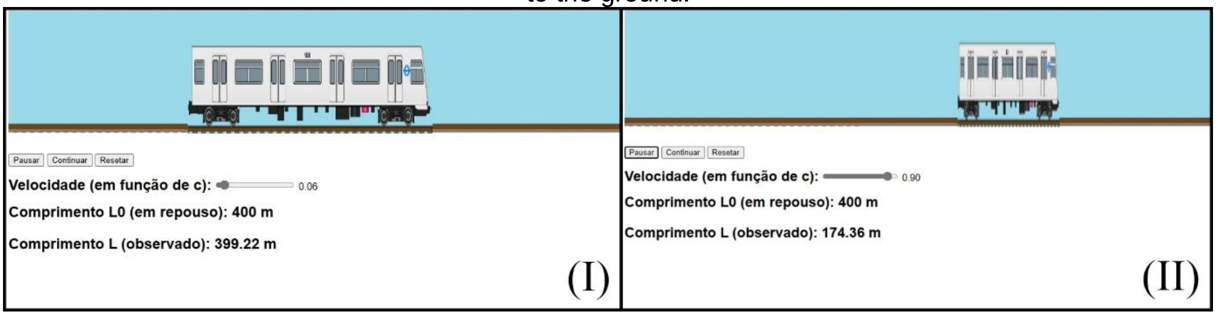
Even after updating the simulations, some compatibility issues were faced during the activities with the students. Therefore, for “Cohort 3”, the “Time Dilation” and “Space Contraction” simulations were redesigned in HTML (Figure 5.7 and Figure 5.8) with the help of *ChatGPT 4.0*. This change greatly facilitated the students’ interaction with the simulations, which could be performed using the Chromebooks available at the school.

Figure 5.7: Updated simulation “Time dilation” showing the train at 0.06c (I) and 0.98 (II) related to the ground.



Source: Author (2024).

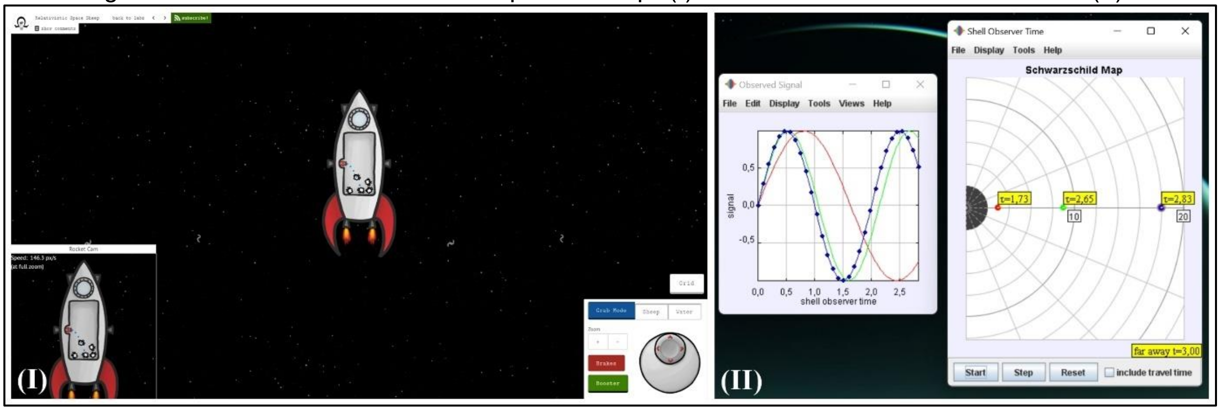
Figure 5.8: Updated simulation “Space Contraction showing the train at 0.06c (I) and 0.98 (II) related to the ground.



Source: Author (2024).

Simulations 5, 6, 7 and 8 – Through the literature review conducted, four simulations addressing aspects related to General Relativity (GR) were selected. The first, called “Relativistic Space Sheep” (Minutelabs, 2014) (Figure 5.9-I), deals with the Equivalence Principle. By accelerating a rocket filled with sheep, it is possible to observe the equivalent effect of a gravitational field on them. The second simulation deals with gravitational time dilation (Christian; Belloni; Cox, 2008) (Figure 5.9-II). Near a black hole, it is possible to observe the passage of time at different distances from it.

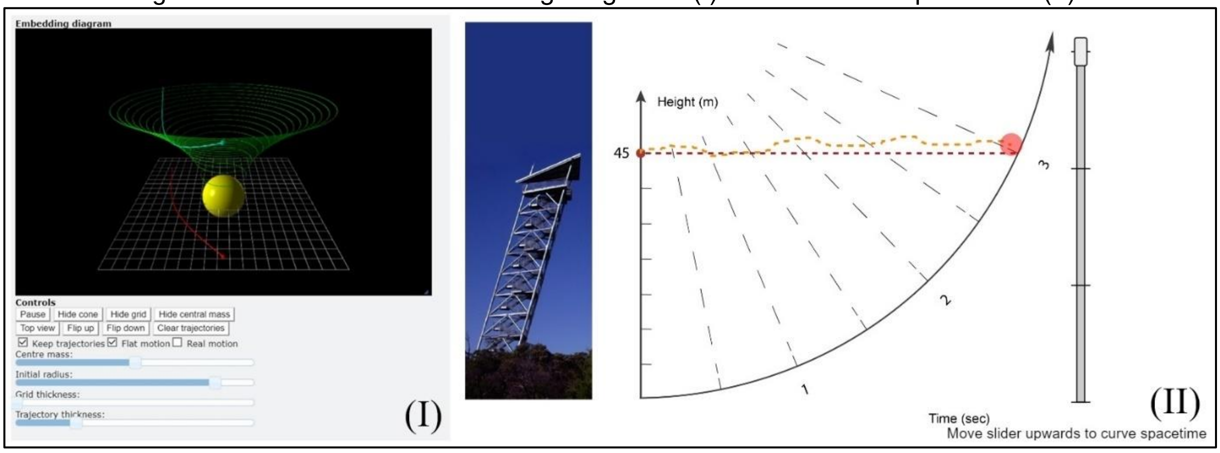
Figure 5.9: Simulation “Relativistic Space Sheep” (I) and “Gravitational Time Dilation” (II).



Source: Adapted from Minutelabs (2014) and Christian, Belloni and Cox (2008).

In the third simulation, embedding diagrams can be observed, showing the two-dimensional curvature caused by a massive object (Ryston, 2019b) (Figure 5.10-I). It is possible to change the mass of the central object and throw a particle to observe its motion. Finally, the fourth simulation is a model available within the virtual environment *ReleQuant* (Kersting, 2019a) (Figure 5.10-II). This model graphically represents the curvature of the time dimension in free-fall situations.

Figure 5.10: Simulation “Embedding Diagrams” (I) and model “Warped Time” (II).



Source: Adapted from Ryston (2019b) and Kersting (2019a).

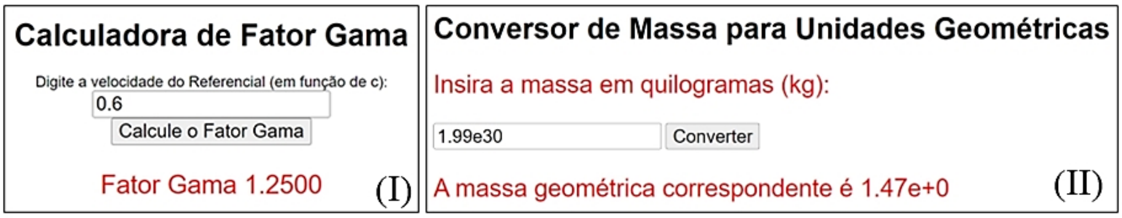
For all simulations, both developed and selected, written guides were elaborated. The guides followed the POE (predict-observe-explain) strategy (Tao; Gunstone, 1999). Through this approach, the student first attempts to predict how a given situation will unfold in the simulation. This situation is then observed, and the student records on the guide what happened. Finally, the prediction and observation are compared in their similarities and differences. All written guides are available at Appendixes B, C, D, E, F and G.

This strategy, based on Piagetian ideas, aims to cause a cognitive conflict during the interaction with the simulation. Therefore, it is essential to emphasize to students that they should not play the simulation before making a prediction and recording it, as this would prevent the reflection proposed. Widely used in the development of simulation manuals, this strategy enables the assimilation and accommodation of concepts during the activity.

Due to software compatibility issues, the two simulations on Galilean relativity and gravitational time dilation were presented by the teacher using a projector. Nevertheless, the same strategy, with adapted scripts, was used. Students interacted directly in pairs with the remaining simulations.

For “Cohort 3”, two new resources were also developed in HTML with the assistance of *ChatGPT 4.0*: a gamma factor calculator and a mass unit converter (Figure 5.11). In the previous edition of the course, it was observed that students had significant difficulty calculating the gamma factor, which distracted them from understand the phenomena due to concerns with the calculation. Therefore, this calculator was developed, which can be used via a smartphone.

Figure 5.11: Gamma factor calculator (I) and mass unit converter (II).



Source: Author (2024).

Furthermore, with the mass converter, it was possible to provide students with mass values in kilograms, allowing them to convert them to geometric units, rather than providing the values directly in these abstract units. This change made the problems to be solved more tangible for the students by dealing with familiar measurement units.

Through the interaction with the presented simulations, students are enabled to use hypercultural mediation. Moreover, as they worked in pairs or groups using the written guide, the social mediation was also enabled.

5.2.3 Practical activities

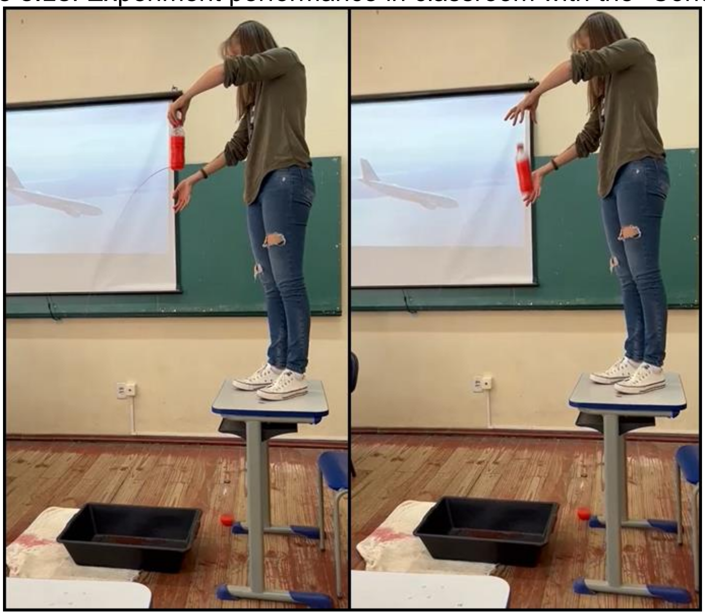
Some experiments and demonstrations were identified in the literature review to be developed during the activities aiming to enable psychophysical mediation. The conceptual ideas under investigation with these activities were the Equivalence Principle, curved spacetime and gravity, and curved three-dimensional spacetime.

Equivalence Principle – For the demonstration of the Equivalence Principle, the first topic of General Relativity addressed, an experiment with a falling plastic bottle was used. A 500 ml plastic bottle with a small hole on the lower lateral side was filled with water.

Figure 5.12: Tests with the plastic bottle.

Source: Author (2024).

Figure 5.13: Experiment performance in classroom with the “Cohort 3”.



Source: Author (2024).

When the bottle is lifted to a certain height, it pours water through the hole; however, when the bottle is released, it experiences a “free fall”, equivalent to the absence of a gravitational field, and the water stops flowing. To facilitate the visualization of the water stream, a dye was used. The tests were conducted with blue dye (Figure 5.12), but for the classes, it was decided to use red dye for greater contrast.

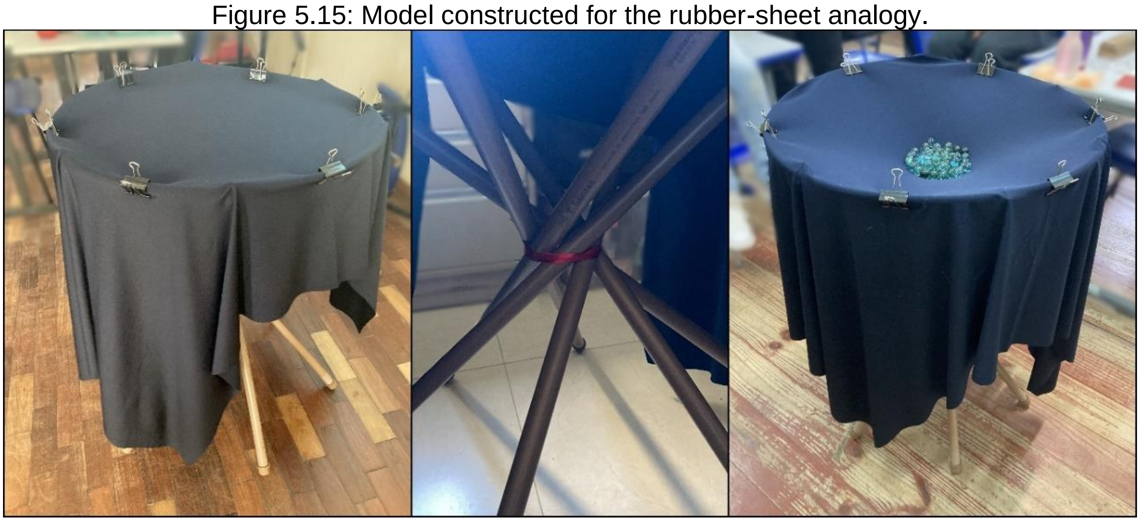
In "Cohort 3", this activity was carried out both in the classroom (Figure 5.13) and in the outdoor area on the school's balcony (Figure 5.14).

Figure 5.14: Experiment performance in a school's balcony with the "Cohort 3".

Source: Author (2024).

Curved spacetime and gravity – Additionally, a model was used for the popular rubber-sheet analogy. The constructed model was based on the one presented by Postiglione and De Angelis (2021a). For this, a 65 cm diameter hula hoop was used to which six 1 m long, and 20 mm diameter PVC pipes were attached, serving as the structure's legs. To fix the pipes to the hula hoop, a 3D printed piece was developed. Representing the fabric of spacetime, a 1.5 m side square Lycra sheet was attached to the hula hoop using paper clips. To represent massive objects, marbles were used (Figure 5.15).

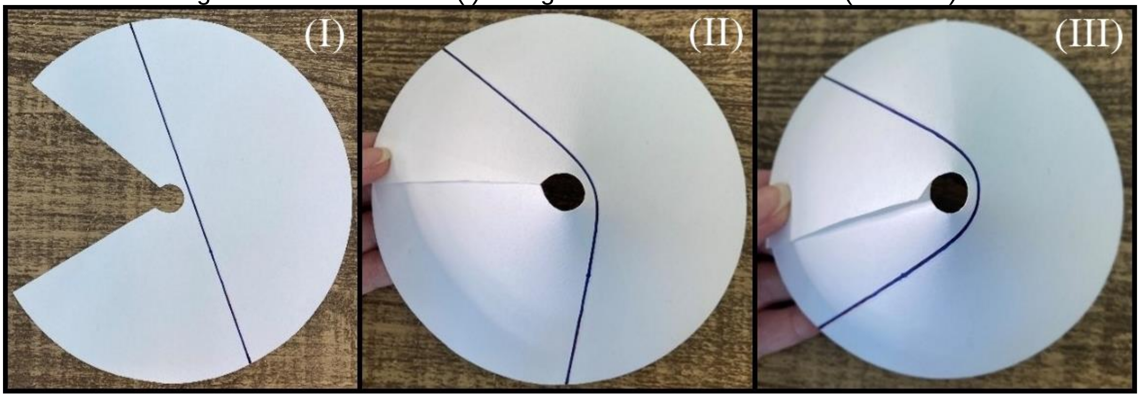
Figure 5.15: Model constructed for the rubber-sheet analogy.



Source: Author (2022).

Furthermore, plain paper cones were organized as presented by (Ryston, 2019a). They were cut out from 180 g/m² paper. Upon receiving the paper, students could draw a straight line and observe how it becomes a curve when forming the cone, working with the idea of geodesics (Figure 5.16).

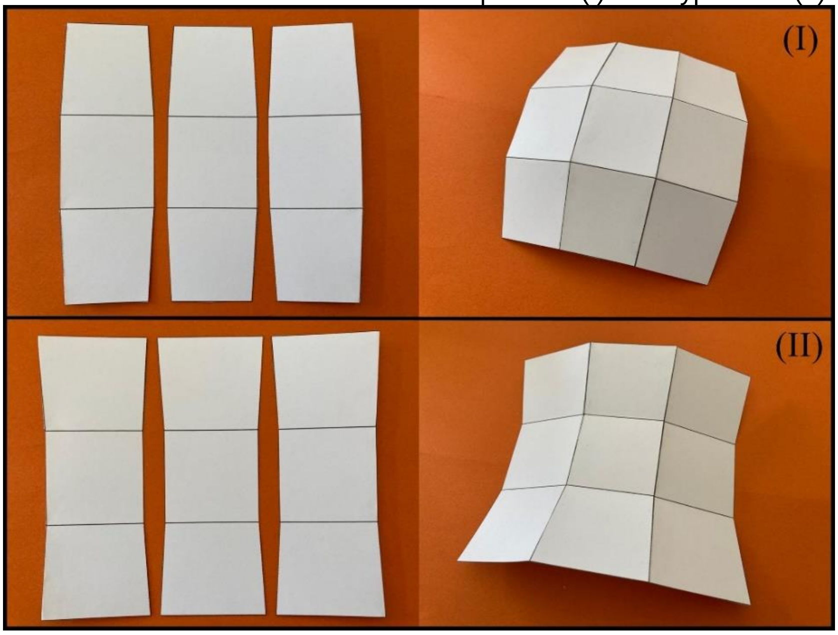
Figure 5.16: Plain cone (I) and geodesics demonstration (II and III).



Source: Author (2022).

Curved three-dimensional spacetime – Finally, the sector models presented by Zahn and Kraus (2014) were constructed to address the three-dimensional curved spacetime. As the both previous models, rubber-sheet and cone, represent two-dimensional surfaces, the usage of sector models allows the extrinsic visualization of three-dimensional curvature.

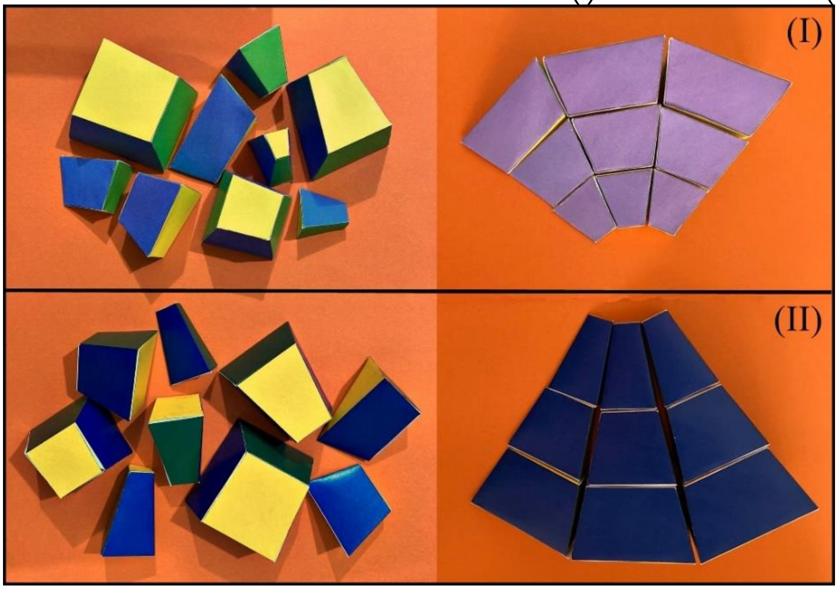
Figure 5.17: Two-dimensional sector models for spherical (I) and hyperbolic (II) surfaces.



Source: Author (2024).

The models provided by Zahn and Kraus (2014) were printed in colour on 180 g/m² paper, cut out, and manually assembled using glue (Figure 5.17 and Figure 5.18). Four sets of models of Euclidean space and curved space were constructed for students to handle.

Figure 5.18: Three-dimensional sector models for Euclidian (I) and non-Euclidian (II) surfaces.



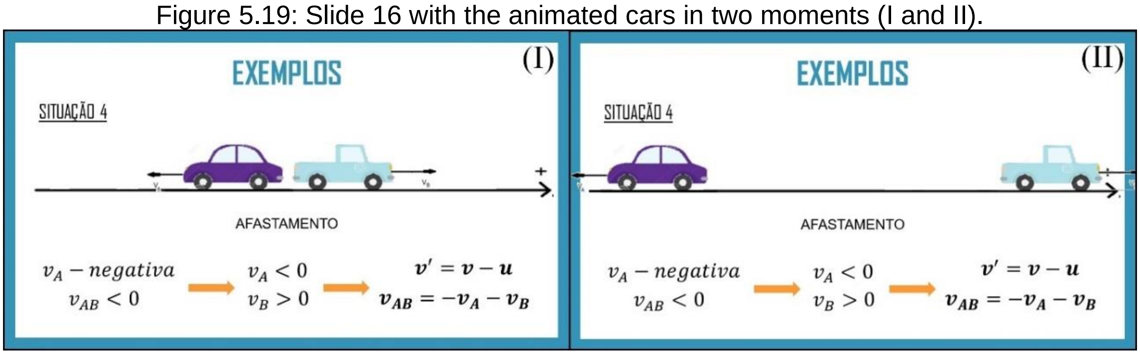
Source: Author (2024).

When the students interact with these practical activities their sensory-motor schemes are stimulated. These physical artifacts helped students' brains to process information through the psychophysical mediation.

5.2.4 Slides presentation

To guide the lessons plan, a presentation with 86 slides was produced (Appendix N), containing varied resources such as videos, GIFs, and animations. Initially, some concepts are reinforced, such as the definition of time and space as measurement results and the concept of inertial reference frame. Then, Galilean Relativity is presented, addressing relative motion and relative velocity. Examples of relative motion are presented with animations of cars (Figure 5.19). The car animations aim to assist students in imagining the situation, allowing them to mentally reproduce it later. The topic is concluded with exercises on relative motion.

Figure 5.19: Slide 16 with the animated cars in two moments (I and II).

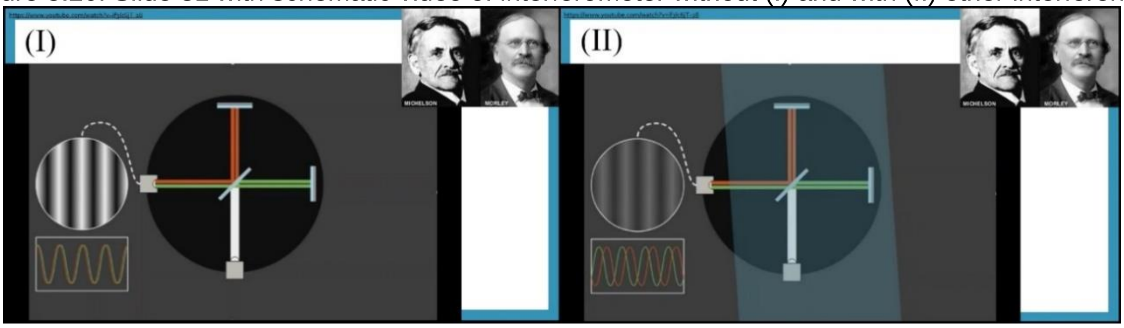


Source: Author (2024).

Subsequently, some historical events that led to the development of Special Relativity Theory are presented: the measurement of the speed of light by astronomer Ole Roemer in the 16th century; the union of electrical and magnetic phenomena with James Clerk Maxwell's equations in the 19th century and their consequences; and the detection of electromagnetic waves predicted by Maxwell experimentally by Heinrich Hertz in 1887.

Given these events, the incompatibility between Electromagnetism and Galilean Relativity is addressed. Three possibilities are presented: the need to adopt a privileged reference frame in electrodynamics (ether), errors in Maxwell's equations, or the reformulation of Galilean Relativity in mechanics. As historically, the first hypothesis was followed, and the Lorentz Transformations with the correction factor γ are discussed. Through a schematic video (Figure 5.20), the Michelson-Morley interferometer of 1887 is presented, highlighting the failure to detect the ether.

Figure 5.20: Slide 31 with schematic video of interferometer without (I) and with (II) ether interference.

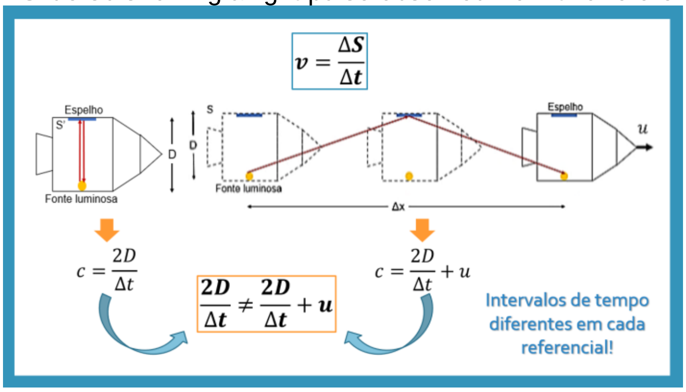


Source: Author (2024).

Next, Einstein's ideas are presented, and Special Relativity Theory is introduced with its two postulates. Some consequences of these postulates, such as the relativity of simultaneity, time dilation, and spatial contraction, are then discussed. To introduce time dilation, for example, a simple situation of the observation of a light pulse from two reference frames was presented (Figure 5.21). Using the average speed, it was shown how the time intervals for the two reference frames must be different to keep the speed of light constant.

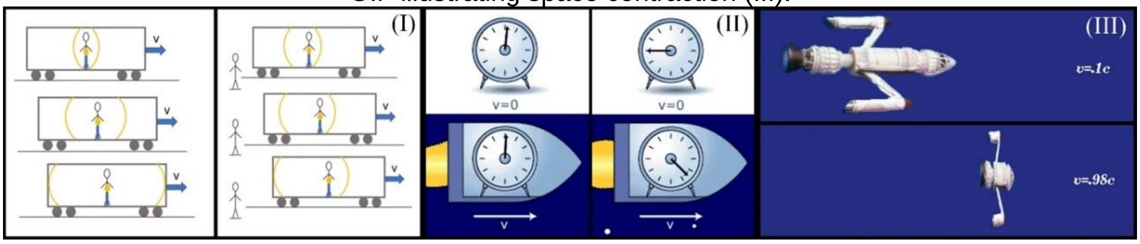
Considering the high level of abstraction of these phenomena, namely, relativity of simultaneity, time dilation, and spatial contraction, images and GIFs were used to illustrate them (Figure 5.22). The equation to calculate time dilation and space contraction were also introduced (Figure 5.23). Finally, an example of relativistic velocity is presented, and six exercises are proposed.

Figure 5.21: Slide 36 showing a light pulse observed from two reference frames.



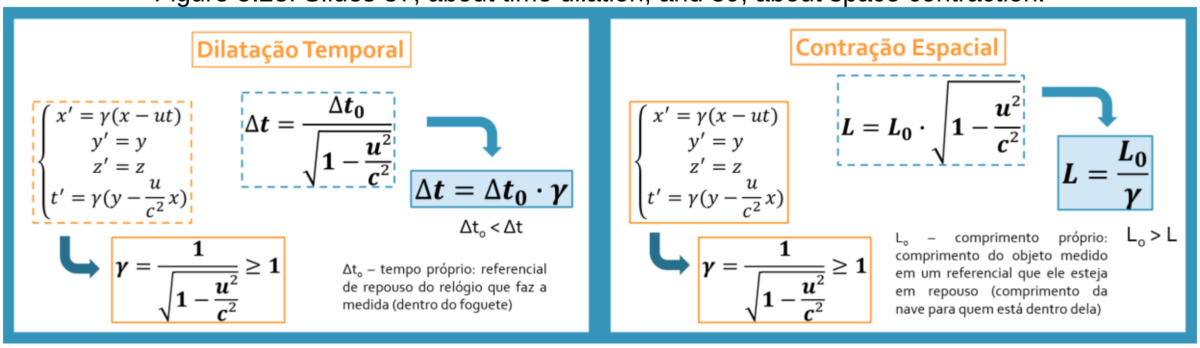
Source: Author (2024).

Figure 5.22: Image illustrating the relativity of simultaneity (I), GIF illustrating the time dilation (II), and GIF illustrating space contraction (III).



Source: Author (2024).

Figure 5.23: Slides 37, about time dilation, and 39, about space contraction.



Source: Author (2024).

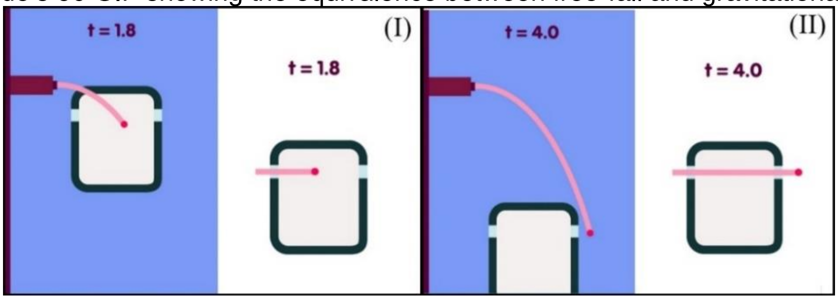
The need to introduce gravity into Einstein's relativity is presented, along with conceptions of gravity throughout history – Aristotle, Galileo and Newton. Einstein's thought experiment of the elevator is discussed, and the Equivalence Principle is introduced. Videos such as "Alice & Bob in Wonderland: What keeps us stuck to the earth?" (Pi, 2010), clips from the European Space Agency's (ESA) Zero-G flight experienced by Stephen Hawking (Figure 5.24), and GIFs are used (Figure 5.25).

The Principle of General Relativity is presented, and the Doppler effect is reviewed briefly to introduce gravitational time dilation. Again, images, GIFs, and a video clip from the Miller's Planet scene in the movie *Interstellar* (Nolan, 2014) are used (Figure 5.26).

Figure 5.24: Screenshots of the Zero-G flight presented in slide 58.

Source: Adapted from Esa (2018); News (2018); Saturnino (2015).

Figure 5.25: Slide's 56 GIF showing the equivalence between free-fall and gravitational field absence.



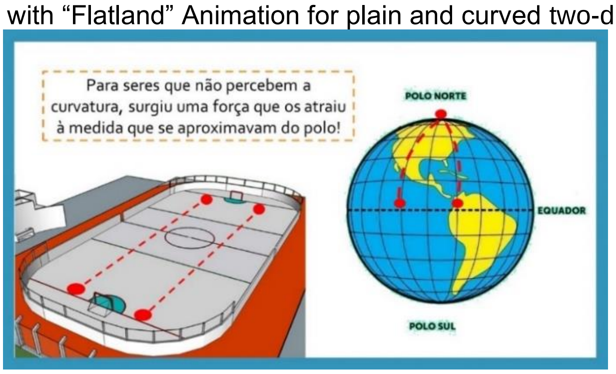
Source: Author (2024).

Figure 5.26: Image from the Interstellar movie's scene.

Source: Adapted from Nolan (2014).

The idea of curved spacetime is then presented, along with the concept of geodesics. It is demonstrated how gravity can be understood as a manifestation of spacetime geometry using the "Flatland" analogy (Thorne, 1994) with an animation (Figure 5.27).

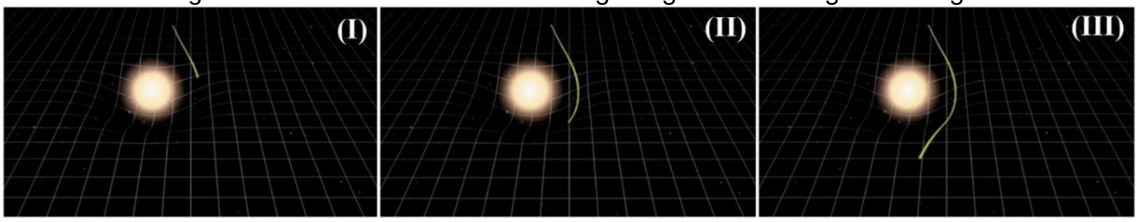
Figure 5.27: Slide 67 with "Flatland" Animation for plain and curved two-dimensional surfaces.



Source: Author (2024).

The rubber-sheet analogy and how massive objects distort spacetime is then discussed. Consequently, the gravitational bending of light is introduced using a GIF (Jackson, 2023) (Figure 5.28). Einstein's Field Equation is introduced to discuss its physical meaning, and then a highly simplified case of the Schwarzschild's Solution is presented to calculate the time dilation and spatial deformation caused by a massive object.

Figure 5.28: Slide's 71 GIF illustrating the gravitational light bending.



Source: Adapted from Jackson (2023).

The prediction of black holes by the Schwarzschild's Solution is discussed, as well as the prediction of gravitational wave propagation in spacetime, as predicted by General Relativity. Five exercises dealing with General Relativity are then introduced. Finally, the main corroborations of the Relativity Theory, namely, Mercury's perihelium precession, Sobral's eclipse of 1919, LHC, GPS, gravitational waves detection by LIGO and first pictures of black holes, are briefly presented as they were the focus of another activity.

More than guide the lessons, the slide presentation also provides different levels of external mediation. The GIFs and animations incorporated in the slides constitute hypercultural external resources, on the other hand, the images and videos enable the cultural mediation. Moreover, considering the discussions promoted by the topics addressed in the slides, the social mediation is also enabled.

5.2.5 Generative Artificial Intelligence

Considering the recent development of Generative Artificial Intelligence (GenAI), two activities using this resource were implemented with the "Cohort 3", covering the Sophotechnic mediation.

5.2.5.1 ChatPDF

The ChatPDF is a free access chatbot that uses the Large Language Model (LLM) GPT 3.5. It allows users to upload PDF files, which are then read by the Artificial

Intelligence (AI), enabling users to ask questions about the document through the chatbot. This resource was used with students to address the main confirmations of the Relativity Theory: the Mercury's perihelion precession, the Sobral eclipse (1919), the *Global Positioning System* (GPS), the Large Hadron Collider (LHC), the first detection of gravitational waves by LIGO (2015), and the first real images of black holes (2019 and 2022).

For each student groups, a topic was attributed, and they received a scientific communication paper on the subject. They were then required to upload the paper, ask questions, and create a digital poster summarizing that confirmation of the theory. The use of articles also enabled cultural mediation.

5.2.5.2 AI image-generator

The Bing Artificial Intelligence image-generator was used by students through their smartphones to generate images for what they imagined for the word "relativity", before and after the course. A user guide for the tool was developed, where students had to register the prompts that they used. Reflecting the fast advancement of AI resources, in the first use, students needed to translate their prompts into English, whereas by the end of the course, it was no longer necessary.

5.3 COHORT 1

What is being considered here as "Cohort 1" consists of the data collection from 14 students conducted in 2019 during the master's research. Activities were developed with the students addressing Special Relativity Theory during the Physics curricular classes. All materials used and activities developed are described in the methodology of the master's dissertation (De Souza, 2021) and are also available in Annex A.

After completing the master's degree, the obtained data were reanalysed with a different perspective than the one adopted in the dissertation, yielding different results. This reanalysis motivated the doctoral research and allowed for the planning of subsequent data collections. For this reason, the data obtained through the master's research are included in this thesis, where they are analysed according to the methodology adopted in the doctoral research that are described below.

5.4 COHORT 2

Considering the large number of planned activities, for “Cohort 2”, it was decided to offer an extracurricular minicourse to students during their free periods. The course, titled “Einstein's Relativity: from GPS to Black Holes”, was advertised in the school and on social media, and students were required to express their interest by filling out an online form. Figure 5.29 shows the posters designed for course promotion.

Figure 5.29: Course’s poster used in school (I) and social media (II).

Source: Author (2024).

Sixteen students started the course, but only eight students completed it, consisting of six girls and two boys. Considering that the course was conducted in the second semester of the year, with the close imminence of university admission exams and the National High School Exam (ENEM), it was expected that a reduced number of students would complete it.

Additionally, the course was affected by several holidays between the classes, which may have impacted student engagement. Table 5.1 presents a summary of the lessons’ organization.

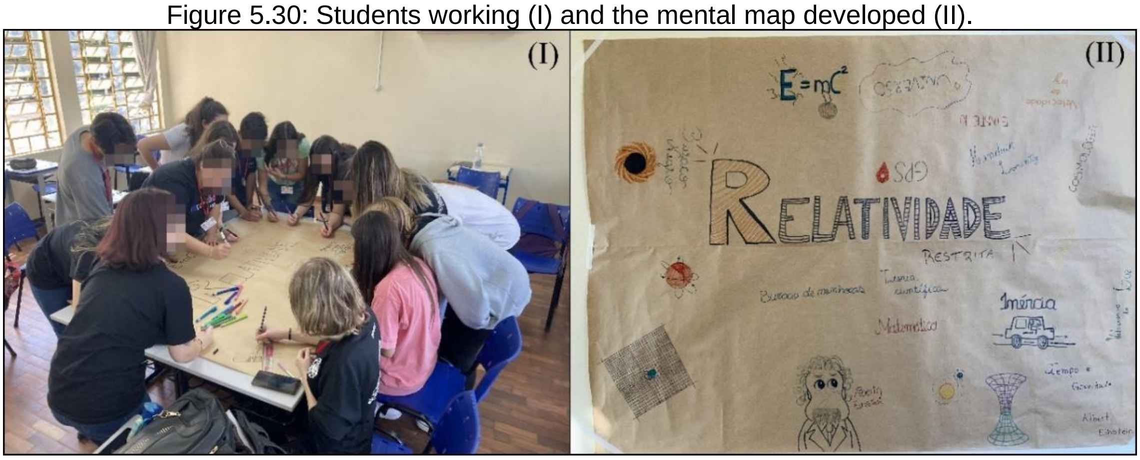
During the first meeting, after introducing the course, students individually responded to a pre-test. Following the test, a collective mind map was created using the word “relativity” on large paper with coloured markers (Figure 5.30). Before concluding the meeting, the concepts of time, space, and reference frames in physics were introduced through the slide presentation.

Table 5.1: Cohort 2 activities.

Class	Date	Subject	Activities
1	24/08	Introduction	Pre-test and collective mental map.
2	31/08	Galileo Relativity	Examples, exercises and correction of the exercises.
3	14/09	Galileo Relativity	Simulations “Ball on the train” and “Cars and plane”.
4	21/09	Special Relativity	Examples and group exercises.
5	28/09	Special Relativity	Correction of the exercises and interaction with the simulations “Simultaneity”, “Time Dilation” and “Space Contraction”.
6	05/10	Special Relativity General Relativity	Completion of simulations activities, gravitational acceleration demonstration with book and paper and plastic bottle experiment.
7	19/10	General Relativity	Interaction with paper cones and sector models.
8	26/10	General Relativity	Simulation “GR Time Dilation”, demonstrations and interaction with the rubber-sheet model, examples and group exercises.
9	09/11	General Relativity	Completion and correction of the exercises and interaction with simulation “Relativistic Space Sheep”.
10	16/11	General Relativity	Interaction with simulations “Curved Spacetime” e “Embedding Diagrams”.
11	23/11	General Relativity (confirmations).	Posters elaboration using the scientific communication papers and post-test.

Source: Author (2024).

Figure 5.30: Students working (I) and the mental map developed (II).



Source: Author (2022).

In the following week, Galilean Relativity was introduced, covering relative movements and velocities through Galilean transformations. Examples were discussed using slide presentations, and a list of exercises was handed out to be completed in groups. At the end of the class, the exercises were corrected in groups.

In the third meeting, activities were conducted using the “Ball on the Train” and “Cars and Plane” simulations. In groups, students received guides with the POE strategy to follow the demonstrations with the simulations through the projector. After completing the activities, some historical events that led to the development of Special Relativity were presented by the teacher.

The Special Relativity Theory was introduced in the fourth class, presenting its two postulates. With the aid of slide presentations, it was discussed how these postulates lead to the relativity of simultaneity, time dilation, and spatial contraction. Space-time diagrams were also presented, and examples of calculating time dilation and spatial contraction were performed, such as muon decay and supersonic jet flight. Finally, in small groups, students received lists of exercises to complete.

In the next class, the exercises were reviewed and corrected. Then, in pairs, students interacted directly with the “Simultaneity”, “Time Dilation”, and “Spatial Contraction” simulations using guides through the POE strategy in the school’s computer laboratory. The simulations presented some compatibility issues, such as with the computes’ screens resolution, changing the position of the elements of the simulation, which delayed the activities.

The completion of the simulation activities occurred in the sixth class. Subsequently, the limits of SR were presented, leading to the need for General Relativity. Historical conceptions of gravity, from Aristotle to Newton, were introduced through the slide presentation. A demonstration of Earth’s gravitational acceleration was performed using a book and a falling paper sheet. The Equivalence Principle was introduced, and an experiment with the falling plastic bottle was conducted in the classroom.

In the seventh class, gravitational time dilation and the concept of curved space-time and geodesics were presented with the aid of the slide presentation. In small groups, students performed an activity with a paper cone and interacted with two- and three-dimensional sector models. At the end of the class, the gravitational bending of light was discussed.

In the following week, the “Gravitational Time Dilation” simulation was used through the projector, and students received guides in the POE strategy in small groups. Subsequently, the rubber-sheet model was used, and some demonstrations were performed regarding deformation and amount of mass, as well as the movement of the balls. Students interacted with the model and asked questions about the analogy.

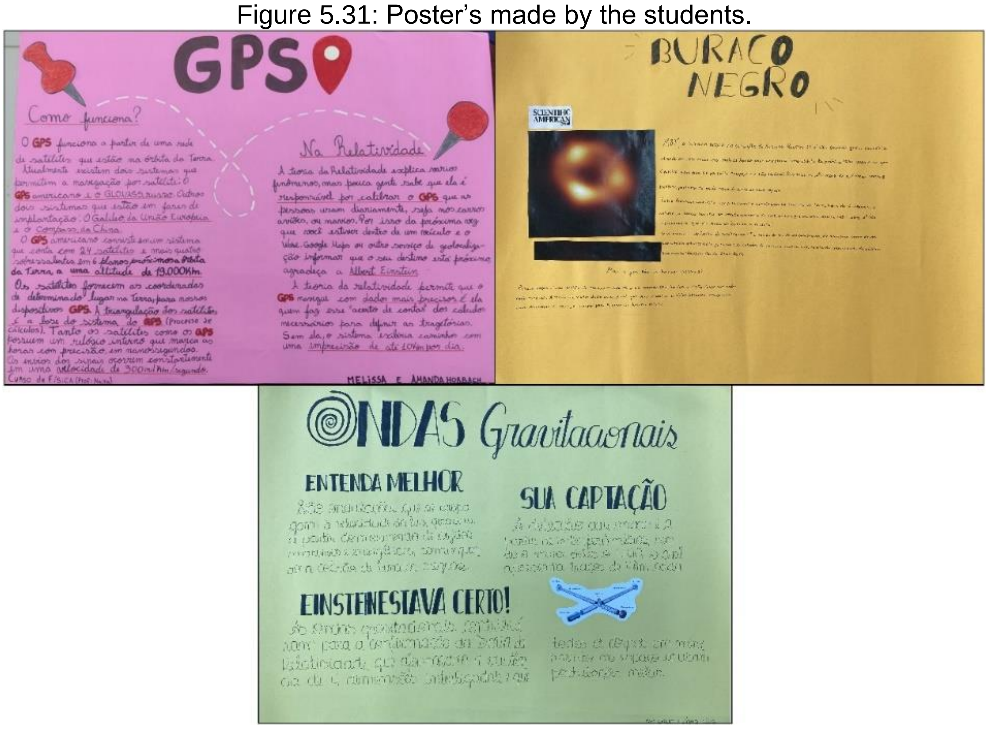
It is worth noting that the limitations of the model were explained, such as the two-dimensional spatial representation and the dependence on terrestrial gravity for the model's functioning.

Next, the meaning of Einstein's Field Equation and the Schwarzschild solution for a simplified case were introduced. The definition of black holes and gravitational waves was presented, and some examples were performed together to initiate a group exercise list. The exercises on GR were completed and corrected in the ninth class. After correcting and clarifying doubts, in pairs, students used the "Relativistic Space Sheep" simulation through a guide with the POE strategy using the school's Chromebooks.

In the next class, in the same pairs, students used the Chromebooks again for activities with the "Warped Time Model" and the "Embedding Diagrams" simulation through guides in the POE strategy. Subsequently, the main confirmations of the Relativity Theory were briefly presented.

In the final meeting of the course, in pairs, students received scientific communication papers about the confirmations presented in the previous class. Each pair was required to read one of the articles and create a poster about it to be exhibited in the school (Figure 5.31). After completing the posters, students individually took the post-test.

Figure 5.31: Poster's made by the students.



Source: Author (2022).

5.5 COHORT 3

The “Cohort 3” consisted of the second edition of the minicourse. As in the first edition, it was offered during the students’ free periods and advertised in the school and on social media. Again, the registration was done through an online form. In this edition of the minicourse, fifteen students started, while ten completed it, six females and four males. It is observed that the percentage of students who completed the course in this edition was higher (67%) compared to 2022 (50%), as expected, as the course was conducted in the first semester of the year.

In this application, there were no holidays between the course classes, favouring the continuity of activities. Moreover, at the beginning of each class, a brief review of the previous class was conducted to clarify doubts and introduce new topics. Based on the observations from “Cohort 2”, some activities were adjusted, and some activities were changed in their order.

Furthermore, with the advent of Generative Artificial Intelligence, one activity was added to the classes, and another was modified. The use of Bing AI for image generation with students was added, and ChatPDF was used for the activity on the confirmations of the Relativity Theory. The Cohort 3 activities are summary on Table 5.2.

In the first meeting, after introducing the course, students individually answered the pre-test. Then, students were guided to the activity with the Bing AI image-generator and received a usage guide, where they had to register their prompts. Students had downloaded the Bing app on their smartphones before the class. Not all of them could access it, and they used a colleague’s device to complete the activity. At the time of this activity the Bing AI only generated images from English commands, so students had to translate their prompts.

In the following week, the introduction of the concepts to be worked on began. First, time, space, and reference frames for physics were defined. Then, the concept of relative movement and relative velocity was introduced, covering Galilean Relativity. Some examples were solved, and then students gathered in groups to solve exercises. At the end of the class, doubts were clarified, and the exercises were corrected in groups.

Table 5.2: Cohort 3 activities.

Class	Date	Subject	Activities
1	26/04	Introduction	Pre-test and generation of the first image with Bing AI.
2	03/05	Galileo Relativity	Examples, exercises and correction of the exercises.
3	10/05	Galileo Relativity	Simulations “Ball on the train” and “Cars and plane”.
4	17/05	Special Relativity	Examples and group exercises.
5	31/05	Special Relativity	Exercises correction and interaction with the simulations “Time Dilatation” and “Space Contraction”.
6	07/06	Special Relativity General Relativity	Gravitational acceleration demonstration with book and paper and plastic bottle experiment.
7	14/06	General Relativity	Simulation “GR Time Dilation”, interaction with paper cones and sector models.
8	21/06	General Relativity	Demonstration and interaction with the rubber-sheet model, examples and group exercises.
9	28/06	General Relativity	Exercises conclusion and correction and interaction with the simulation “Relativistic Space Sheep”.
10	05/07	General Relativity	Simulations “Curved Spacetime” and “Embedding Diagrams” and activity with ChatPDF.
11	12/07	General Relativity (confirmations)	Conclusion of ChatPDF activity, discussion about exams’ questions, generation of image with Bing AI and post-test.

Source: Author (2024).

In the third class, the concepts of relative movement and velocity were reviewed. Then, the “Ball on the Train” and “Cars and plane” simulations were used through the POE strategy. In small groups, students received guides to follow the demonstrations on the projector. After using the simulations, the historical events that led to the development of Special Relativity were addressed.

Using slide presentations, more historical events were discussed, and the Special Relativity Theory was introduced in the fourth class. The phenomena of relativity of simultaneity, time dilation, and spatial contraction were presented. Some examples were performed together, such as muon decay and supersonic jet flight. Then, students received exercises to be completed in groups (Figure 5.32). Students had access to the gamma factor calculator in this edition of the course, facilitating the completion of the exercises.

Figure 5.32: Cohort 3 students solving exercises in groups.

Source: Author (2023).

The exercises were finalized and corrected at the beginning of the fifth meeting. Then, using the school's Chromebooks, students used the "Time Dilation" and "Spatial Contraction" simulations in small groups. Each group received a usage guide through the POE strategy. The use of Chromebooks was possible since the two simulations were adapted to HTML in this edition of the course.

In the sixth class, after a brief review of Special Relativity, its limitations were discussed, mainly the need to include gravity in the theory. Then, some historical concepts of gravity were presented. As in the previous edition, the Equivalence Principle was introduced, and the experiment with the plastic bottle was conducted in the classroom. From students' ideas, the experiment was repeated on the school's balcony. After the demonstrations, the Doppler effect for gravitational time dilation was briefly introduced.

In the following week, the phenomenon of gravitational time dilation was reviewed using the "Gravitational Time Dilation" simulation on the projector. In small groups, students received written guides in the POE strategy to follow the demonstrations. Then, using the slide presentation, the idea of curved space-time and geodesics was introduced. At the end of the class, in small groups, students performed the activity with the paper cone and sector models (Figure 5.33).

The eighth meeting began with a review of curved space-time through demonstrations and interaction with the rubber-sheet model. Then, Einstein's Field Equation and the Schwarzschild's solution were introduced. Some examples were solved together, and students started the solution of exercises in small groups. In this edition of the course, the mass converter to geometric units was used by students, and they could receive mass values in kilograms.

Figure 5.33: Cohort 3 students interacting with the sector models.

Source: Author (2023).

In the next class, the exercises were completed by the students and corrected in groups. Some doubts were clarified, and then students used the “Relativistic Space Sheep” simulation in pairs through the Chromebooks. Each pair received a written guide in the POE strategy.

In the same pairs, students interacted with the “Curved Spacetime” and “Embedding Diagrams” simulations in the next class. After completing the activity, the confirmations of the Relativity Theory were presented, including the recently detected NanoGrav. Still in pairs, students received scientific communication papers about the confirmations of the theory. Each pair uploaded their article to ChatPDF and had to ask questions to understand it and create a digital poster.

In the final meeting of the course, some pairs were finalizing their posters. As they finished the activity, students moved on to generating images with Bing AI. At the end of the course, it was no longer necessary for the prompts to be in English, so there was no need to translate them. After completing these activities, a question from the 2022 ENEM and one from the 2023 UFSM admission exam, which dealt with black holes, were discussed. Finally, students individually responded to the post-test.

5.6 INTERVIEWS

Following the activities, all students were interviewed. The student interviews were scheduled during non-class hours and were conducted by the researcher at the school. The interviews were individual, with only the researcher and the student present. Each complete interview was video recorded for subsequent transcription and analysis. The recordings were made using a mobile phone for two main reasons: (1) the availability and accessibility of the resource; and (2) the minimal potential impact on the activities, as the students were familiar with the equipment and could easily ignore it.

The interviews were based on the pre-test and post-test questionnaires for each student. For the students in "Cohort 3" additional questions were asked about their generated images using Bing AI before and after the activities. During the interviews, students had access to their tests, simulation user guides, and exercise lists. Moreover, they could access the simulations or slides if requested.

The interviews were conducted following the *Report Aloud* protocol (Trevisan; Serrano; Wolff; Ramos, 2019), an adaptation of the *Think Aloud* protocol (Van-Someren; Barnerd; Sandberg, 1994). Through *Think Aloud*, the interviewees literally think aloud as they solve a problem. This method allows for understanding what the students are thinking when responding to questionnaires and interacting with a simulation, for example. As they form their response, they describe their thought process to the interviewer.

However, this technique can cause interference on the participant's cognitive process. Some students may feel pressured when having to solve a problem in front of the interviewer. The Report Aloud method overcomes this obstacle. Instead of the interviewee reporting their thoughts while performing an activity, they do so afterwards. This way, there is no interference with the student's cognitive process while performing the task. However, it is worth noting that the accuracy of the report is reduced, as it relies on the interviewee's memory. Therefore, it is essential that the interval between the activities and the interviews is not too long.

Given that students were questioned about activities they had previously performed, the Report Aloud protocol was primarily used. However, during the interview, new questions were proposed when opportune, and the student responded using the Think Aloud approach. Both protocols maintain a constant dialogue between the interviewer and interviewee, seeking to understand the student's reasoning processes while performing or having performed the activities.

Therefore, during the interview, students were constantly encouraged to externalize their reasoning processes to the best of their ability. Obviously, stimuli were used with great caution to avoid any influence on the students' responses. Through the interview it was possible to identify those external mediation levels that mostly influenced students' conceptual understanding and mental representations.

5.7 DATA ANALYSIS METHODOLOGY

Since different formats of data were obtained in this research, both a gestural and a textual and verbal analysis were performed. The analysis methodologies used are presented in this section.

5.7.1 Depictive Gestural Analysis

All interviews were fully transcribed from the recordings to conduct the Depictive Gesture Analysis (Clement; Steinberg, 2002; Monaghan; Clement, 1999; Stephens; Clement, 2015) of each interview. This qualitative analysis is based on the premise that there is a connection between the depictive gestures performed by students and the mental representations present in their cognitive structure.

In this analysis, gestures are considered as expressions of mental processes. Therefore, they play the role of a "parallel discourse" that happens simultaneously with verbal discourse. According to Monaghan and Clement (1999), these gestures are a way for the student to externalize what they are thinking at the moment, serving as an indicator of their mental images and simulations.

Although depictive gestures are related to verbal discourse, they provide different information. Gestures are a language of their own, expressing what the student may not be able to convey through verbal language. Thus, by analysing and interpreting these gestures, it is possible to identify information not accessible through discourse or writing. Therefore, it is essential to analyse beyond what the student is saying or explaining, understanding what they are imagining.

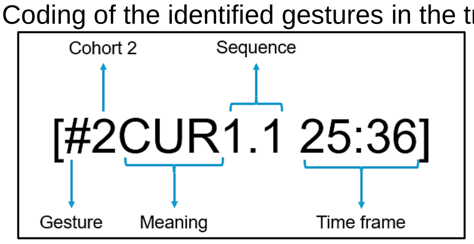
In this way, identifying patterns of gestures and their relationship with the knowledge present in the student's cognitive structure is crucial. Identifying depictive gestures allows for investigating whether students have internalized an external resource through their interaction with it. Gestures can indicate whether the student is accessing this resource “off-line”, i.e., without external stimulation, through the constructed mental representation.

Performing a depictive gesture, which is a representational gesture, is different from gesticulating (Stephens; Clement, 2010). Therefore, to identify depictive gestures, two main indicators are used. Firstly, representative hand movements (“representative gestures”) are identified, indicating forms, locations, or movements of objects. Then, imaginative reports are identified, where the student mentions that they “imagine” or “see” a particular scene or object to answer a question. Thus, students who performed representative gestures during the interview were encouraged to describe what they were imagining.

There are two basic types of gestures: static and dynamic (Clement and Steinberg, 2002). Static gestures are related to mental images, representing static situations. They can be identified when the student uses their hands to represent a ball, for example. On the other hand, dynamic gestures are linked to mental simulations, representing situations in motion. If the student uses their hands to represent the movement of a thrown ball, they are performing a dynamic gesture.

Therefore, after transcribing the interviews, all depictive gestures performed by all students during the interviews were identified. Then, the gestures were catalogued and coded. Each gesture was recorded through a sequence of screenshots from the interview recording, organized and saved as images. In the transcriptions, each gesture was indicated by markers “[#]” followed by its code and the moment in the video when it was performed (Figure 5.34).

Figure 5.34: Coding of the identified gestures in the transcriptions.



Source: Author (2024).

To provide reliability in this analysis, the findings were systematically compared with other studies as well as discussed among the research group members. Moreover, when students performed depictive gestures during the interview, they were questioned directly about what they were imagining. This action provided a confirmation from the interviewed about that gesture interpretation.

5.7.2 Textual and Verbal Analysis

During the interview, conducted through the Report Aloud protocol, the interviewer constantly questioned the students about what they were thinking or imagining. In general, when the students used mental representations, images, or mental simulations, depictive gestures were performed. In this case, the Depictive Gesture Analysis was conducted.

However, in moments when the student did not use depictive gestures or reported imagining something, the investigation of propositional mental representations was pursued. To do so, the verbal reasoning and the way students expressed themselves through discourse during the interview were also analysed. For this, the consistency of the discourse and the meaning of the reported sentences were evaluated.

It is worth noting that a dialogue is never analysed directly, but rather through written transcriptions. Therefore, some precautions must be taken during transcription. Sentences that are perfectly coherent and intelligible in discourse can become confusing in their transcription (Lemke, 2012). While making necessary adjustments to the transcription, it is essential to maintain the meaning of the discourse, which depends on the context in which it is inserted.

The analysis of discourse is based on comparisons. Lemke (2012) highlights that the meaning of verbal data is obtained in relation to the context in which it was produced and the context in which it is used. Thus, any sentence or statement made by a student during the interview can only be understood and analysed by considering its context, such as the question asked by the researcher or the activity to which the statement refers.

Analysing discourse is an interpretive activity that produces the best results when accompanied by information about the context of the discourse. Although discourse analysis does not provide much generalizable information, it provides ways to analyse and understand what occurs in a particular case being analysed. It is common in education research to use students' reports to identify mental representations, as they are connected to the subjective experience of perception (Kosslyn; Thompson; Ganis, 2006). Therefore, we acknowledge the difficulty and limitations of this type of analysis, as it is based on behavioural indicators of the subjects.

6 RESULTS COHORT 1

As mentioned earlier in Chapter 5, Methodology, the Cohort 1 consisted of re-analysing the data obtained during the master's research. The pre-tests, post-tests, and interviews of the 14 students were re-examined, both textually and through Depictive Gesture Analysis (see Chapter 5, Section 5.7.1). Although gesture analysis was conducted during the master's research, the investigation focused on identifying students' mental simulations and conceptual profiles.

Upon re-examining the results, it was found that students did not use mental simulations for some situations. Consequently, the data were re-analysed, investigating not only mental simulations but also all types of mental representation that students may have developed, both pictorially and propositionally (as presented in Chapter 4, Section 4.1).

This re-analysis of the data presented a new perspective, which guided the planning of data collection activities for the doctoral research (Cohort 2 and Cohort 3). In addition to including other formats of mental representations in the investigation (pictorial or propositional), the research also sought to identify patterns in the use of these formats in relation to the concepts worked on (for example, time dilation and space contraction).

In this sense, the students were categorized into three groups based on the analysis conducted: satisfactory understanding and consistent mental representations (4); partial understanding and developing mental representations (5); and limited understanding and inconsistent mental representations (5).

Within each group, the conceptions and mental representations of the students for each key concept are analysed. Since Cohort 1 dealt with Special Relativity, the conceptions and representations of students regarding time dilation and space contraction were examined. The cases of six students, two from each group, are presented here.

6.1 SATISFACTORY UNDERSTANDING AND CONSISTENT MENTAL REPRESENTATIONS

The students Finn and Lily presented a good level of comprehension both for time dilation and space contraction, as well as similar mental representations.

6.1.1 Time Dilation

When discussing questions 5 and 7 of the tests, which dealt with time dilation, student Finn demonstrated a good understanding of the phenomenon. The two questions presented the same situation: “Consider two friends with the same age. One of them goes for a travel in a boat, while the other remains in the city where they both live. For the friend who stayed, the one who travelled returns to their city 1 year after departure”. The question asked about the time taken for the friend who travelled when the ship was travelling at 56 km/h (question 5) and 0.7c (question 7).

Finn responded correctly “less than a year” to question 7 and “equal to a year” to question 5. While explaining his responses, he demonstrated a good understanding of the gamma factor, relating the speed of the reference frame to the effect of time dilation. As he explained:

> […] I thought 56 km/h is irrelevant to change the time, now, getting close to the speed of light, it’s a much higher speed, so it can decrease the time. That’s why I changed, I had marked here [question 5] less than one year, but I got here [question 7] 0.7c, so it’s a much higher speed, that will be relevant, and 56 km/h won’t.

Finn recognised that at a speed of 56 km/h, the difference in time interval between the two reference frames would be “irrelevant”, while at 0.7c, it would be significant, suggesting that lower speeds would not cause a significant change in measurements. His explanation suggests that the student developed a conceptual understanding of time dilation.

However, Finn did not present indicators of mental simulation use for time dilation; the student described the situation verbally, not performing descriptive gestures and not reporting imaginative mental processes. Additionally, when asked what he remembered to answer the question, Finn stated:

> I remembered the twins, that was what struck me most. Because that made me understand better, I thought about the story, if they’re two people on the same planet, but twins are almost the same person, with the same characteristics, if they are in different places [reference frames] and time will pass in a different way, so that’s what will happen.

Therefore, the student demonstrated an understanding of time dilation without using visual resources. Instead, Finn used similar situations involving time dilation to build a verbal argument, as he mentioned “I thought about the story”. Therefore, it is likely that he developed a mental representation in a different format, without resorting to images.

As the situations reported by Finn were discussed during classes through exercises in pairs, possibly social mediation allowed him to process external information and understand the phenomenon of time dilation. His response indicated that, through social mediation, Finn was able to develop *drivers*, in the form of propositional mental representations, which enabled him to understand time dilation.

Another student, Lily, also presented a good understanding of time dilation. Exploring her answer to question 7, she stated that “less time passed” for those who travelled and that “time would be different”, suggesting that she accepted the validity of the phenomenon, not just relating it to perception. Additionally, Lily correctly related time intervals to the speed of the reference frame: “as the speed was so high, [space]ship’s time would be different for the person who stayed [on the land]”, she stated.

In another moment of the interview, while discussing the “Time Dilation” simulation, Lily was asked about an intermediate speed and provided further evidence of this understanding, stating: “I think it also would have a difference, but not so much. As slower the speed, less is the difference”. This excerpt suggests that Lily was able to understand the role of the gamma factor in time dilation.

Following the interview, Lily was asked if she imagined something related to the phenomenon of time dilation. At this point, she explicitly stated that she could not “see” what happened in the phenomenon: “I cannot imagine, like, it’s not very concrete for me. But I can understand, like, this part, but I cannot see what’s happening”. This report, aligned with the fact that Lily did not perform descriptive gestures, indicates the absence of images or mental simulations for the phenomenon of time dilation.

However, as Lily demonstrated a reasonable conceptual understanding of time dilation, possibly she was able to develop another type of mental representation, possibly in a propositional format, similar to Finn. Additionally, similarly to him, Lily mentioned remembering discussions during classes:

> I think [I remembered] the travels you mentioned, if someone took a rocket and went to the Moon, or any other place, at a very high speed, the person would come back younger than the person who stayed on Earth. For thar person who went to the Moon, less time would pass.

Observing this report, possibly Lily used social mediation to develop *drivers*, in the form of propositional mental representations, helping her to process information related to time dilation. Therefore, she did not demonstrate internalising external visual resources used to deal with this phenomenon but developed propositional structures (logical sentences) from class discussions.

It is worth noting that the situations proposed involving time dilation only involved questions about observations made from a stationary reference frame relative to the ground. Even students with satisfactory understanding when explaining their responses did not consider the observation from a reference frame in motion relative to the ground.

This unilateral perception of the phenomenon by students has been already observed by Alstein, Krijtenburg-Lewerissa and Van Joolingen (2021). Although this perception shows a limitation in student understanding, it is understandable considering the level of instruction and brief contact with Special Relativity. Therefore, a correct analysis, even with a unilateral perception, was considered here as satisfactory understanding.

6.1.2 Space Contraction

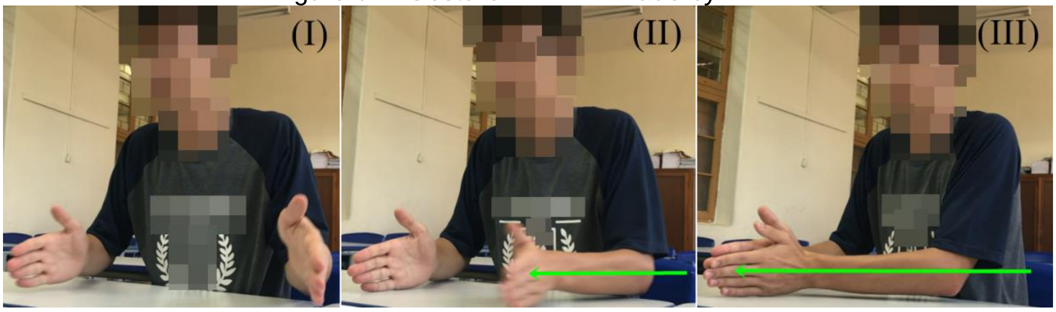
Questions 1 and 3 of the tests dealt with the phenomenon of space contraction. Both questions proposed the same situation: “Two trucks are on the same road, moving in opposite directions, as shown in the figure. Truck 2 is stationary due to an engine problem. The distance between them, measured by Truck 2, is 100 m”. The question asked about the distance measured by the moving truck when it was travelling at 80 km/h (question 1) and 0.7c (question 3).

Discussing question 3 of the test, Finn and Lily demonstrated a good understanding of space contraction. Finn exhibited a particularly developed understanding with a clear mental simulation for space contraction: a spaceship moving and reducing its length as it moves faster.

Curiously, Finn marked the option “equal to 100 m” on the pre-test and “greater than 100 m” on the post-test – both incorrect. However, during the interview, he corrected his answer, stating that “the higher the speed, the smaller the distance [between the trucks]” and that the correct answer would be “less than 100 m”. Additionally, Finn described his mental imagery processes:

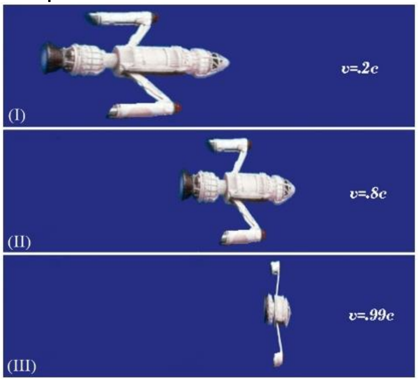
> I remembered the slides, there was a spaceship. What most impressed me was that the spaceship was [#1NAV4 17:50] decreasing in [length] ... I believe I thought about it, because then [#1NAV4 18:14] its size decreased the faster it went, so the faster it [truck] was, the shorter the path [#1NAV4 18:20].

Figure 6.1: Gesture #1NAV4 made by Finn.



Source: Author (2024).

Figure 6.2: GIF presented in the slide 50 in three moments.



Source: Author (2024).

Both Finn's depictive gesture #1NAV4 (Figure 6.1) and his report are indicators that he developed a clear mental simulation for the proposed situation. He placed his two hands in front of his chest, and then brought his left hand closer to his right hand, as gestural evidence of the mental simulation. The spaceship described by Finn and his gesture #1NAV4 are notably similar to a GIF used in the slides (Figure 6.2); during the interview, he confirmed this. The student was able to transpose the situation presented by the GIF, from the decrease in the size of the spacecraft, to the path of the lorry.

Therefore, Finn was able to construct a mental simulation (internal mechanism) and access it without the external stimulation from the GIF. This mental simulation consists of the *driver* he developed, that aided his reasoning process to solve a problem proposed in the test. Finn understood space contraction at a good level and was confident in the representations used to correct an earlier incorrect answer.

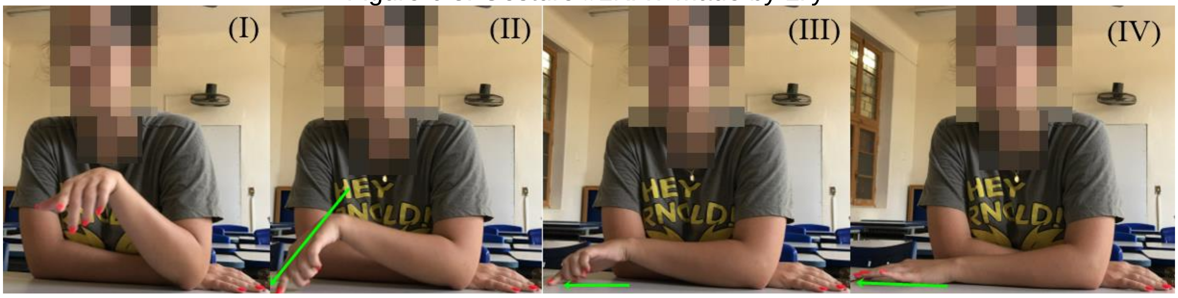
Finn changed the situation viewed earlier in class, through hypercultural mediation, to the situation proposed in the test, as he said, "I imagine the path literally decreasing, as if was in the GIF, but the path and not the spaceship", correctly responding to the question. This description reinforces the existence of mental imagery processes and mental simulations, which were constructed by interaction with a hypercultural resource (GIF), for different contexts and situations.

When asked about the scenario with a lower speed for the truck (0.2c), Finn stated that the road would have an intermediate size compared to questions 1 (80 km/h) and 3 (0.7c), suggesting an understanding of the gamma factor and its connection with the speed of the reference frame. Additionally, as the student stated, “the distance will be smaller” and “the path would be smaller”, it is possible to infer that Finn accepted the validity of space contraction, not relating it to perception effects.

Lily showed similar conceptions, responding correctly to question 3 and explaining that “speed could distort the distance”, suggesting that she also did not relate the phenomenon to perception. Additionally, Lily demonstrated an understanding of the gamma factor and its connection with the speed of the reference frame. When asked about a situation involving a lower speed (0.2c) for the truck, she recognised the difference in effects. Moreover, Lily stated directly remembering the same animated GIF from the slides mentioned by Finn, stating:

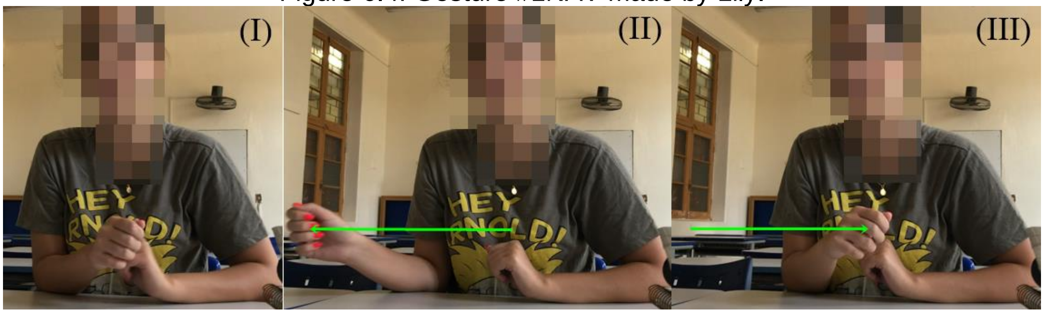
> I think the same thing would happen, because it’s a very high speed, even if it’s 0.2[c]. I think it would be very high and, you know, the difference in distance wouldn’t be so big, but I think there would be a difference. [...] I don’t remember exactly what it was, you showed a slide that was distorted [#1NAV 16:34] I think. I don’t know if it was a rocket, what it was. [...] That, as you moved it, [#1NAV2 2x 16:41] the thing was distorted like that.

Figure 6.3: Gesture #1NAV made by Lily.



Source: Author (2024).

Figure 6.4: Gesture #1NAV made by Lily.



Source: Author (2024).

Lily performed the depictive gesture #1NAV (Figure 6.3), where she moves her left index finger to the right, indicating something moving, and the gesture #1NAV2 (Figure 6.4), moving her right hand to the left, right, and left again, indicating the change in the size of the rocket as it travels. This gesture resembles Finn's gesture #1NAV4 and, aligned with her discourse, are indicators that Lily developed a mental simulation for space contraction: a rocket with its size changed by speed. As the student mentioned the slides and confirmed during the interview, she used the interaction with the hypercultural resource (GIF) to construct *drivers* in the form of a pictorial mental representation and used it to solve new problems proposed.

What is particularly noteworthy in this case is that Lily did not directly remember the nature of the object, but rather the fact that, regardless of the object, the result would be the space contraction, a phenomenon represented by her mental simulation. Like Finn, Lily was also able to access and use her mental representation, in the form of a mental simulation, in different situations to solve different problems.

6.2 PARTIAL UNDERSTANDING AND DEVELOPING MENTAL REPRESENTATIONS

Elle and Iris presented a good level of understanding for time dilation, with results similar to Finn and Lily for this phenomenon. However, regarding space contraction, these students presented a limited comprehension with undefined mental representations.

6.2.1 Time Dilation

Regarding question 7 of the test, where a friend is traveling on a boat and the other is waiting on land, Elle stated that the velocity "would change time" and that the time interval "was less than one year for the person who was travelling". These assertions suggest that the student considered time dilation as a tangible phenomenon, rather than a perceptual issue associated with the psychological concept of time.

In another segment of the interview, Elle was asked about a speed of 0.5c for the ship and stated that the time interval "would be a little different [from 0.7c, question 7], but not like at 56 [km/h, question 5]". The student's response implies an understanding of the role of the gamma factor in introducing a gradual difference in time measurements with the speed of the reference frame.

Although Elle answered correctly to the question, stating that less time would pass for those travelling (the question considered the reference frame of land), she only provided a verbal explanation for it. The student did not perform depictive gestures or report imaginative mental processes. Therefore, it was not possible to find indicators of the use of pictorial mental representations. Since Elle presented a correct verbal explanation for the phenomenon, this indicates that the student possesses conceptual understanding of it. This result suggests the use of a propositional mental representation, without resorting to images.

Another student, Iris, presented similar results when discussing the same question on the test. She stated that “time passes faster for those who are there, outside [the boat], not travelling”, according to a reference frame on land. This declaration suggests that the student accepted the validity of time dilation, rather than relating it to perception. Furthermore, when questioned about a change in speed to 0.5c, Iris confirmed that “there would be a difference, but not much”, implying an understanding of the effects of speed on measured time intervals.

Like Elle, Iris did not perform depictive gestures or report the use of imaginative mental processes during her explanations. Therefore, it was not possible to identify indicators of the presence of pictorial mental representations. Considering that Iris was able to explain correctly time dilation verbally and answer to the question, it is possible to assert that she possesses conceptual understanding of the phenomenon. Therefore, like Elle, Iris was able to construct a propositional mental representation.

Additionally, when asked what she remembered when answering to this question, Iris mentioned the exercise on the twins’ paradox discussed in class, stating that she remembered the “part of the calculations” and “the exercises”. Her response indicates that she did not rely solely on external visual resources to assimilate the concept, but also learned from class discussions. The students developed and discussed the exercises in pairs during classes, indicating that social mediation assisted Iris in processing information. Therefore, through social mediation, she was able to develop *drivers* in the form of propositional mental representations.

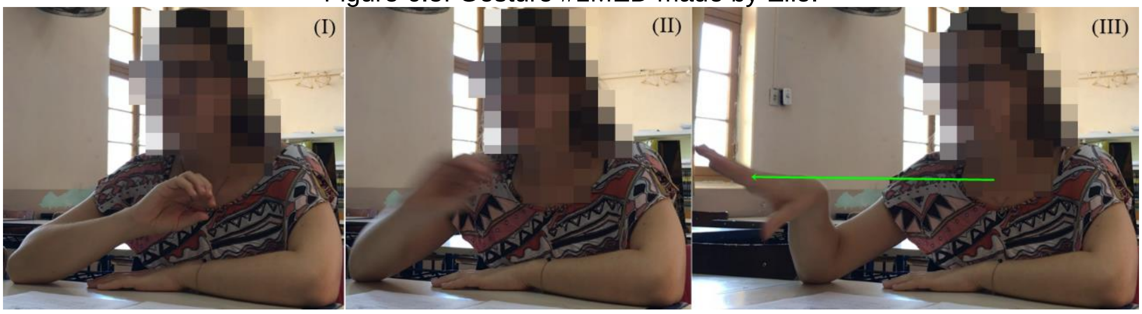
Like the students with satisfactory understanding, Elle and Iris explained their responses from a reference frame stationary to the ground. As mentioned before, this unilateral perception of time dilation is common among school students (Alstein; Krijtenburg-Lewerissa; Van Joolingen, 2021).

6.2.2 Space Contraction

Although Elle and Iris demonstrated a good understanding of time dilation, they did not present the same result for space contraction and were, therefore, considered to have “partial understanding”. Regarding question 3, about the distance measured by a moving truck, even though Elle responded correctly “less than 100 m” in the post-test, she stated that “there would be a difference in perception”, relating the phenomenon to the inability to see the object correctly.

Furthermore, the student mentioned imagining “observing it [the truck] very fast [#1MED 18:35], as if it was a flash”. This report, along with Elle’s gesture #1MED (Figure 6.5), indicate the presence of an undefined mental image of something moving at high speeds. The student gestured quickly with her right hand to indicate the movement of the truck at 0.7c, suggesting that it could not be perceived clearly, “appearing” smaller. This gesture and Elle’s report reflect her difficulty in imagining this situation and recognizing the validity of different measurements between different observers.

Figure 6.5: Gesture #1MED made by Elle.



Source: Author (2024).

When asked if she would change her answer if the truck were moving at 0.5c, Elle stated that “it would be fast too”, suggesting that she did not perceive any difference compared to the speed of 0.7c. Therefore, probably she did not understand the connection between the gamma factor and the reference frame’s speed.

Considering her report, possibly Elle related the phenomenon of space contraction to her daily experiences observing moving vehicles (Figure 6.6), using psychophysical mediation. This external mediation cannot explain or represent relativistic effects correctly, and therefore was not efficient in assisting the student in processing information. This difficulty in associate the relativistic phenomena with daily life is common among students when learning Special Relativity (Yavaş; Kızılcık, 2016).

Figure 6.6: Example of a car moving fast.

Source: Bontan (2018).

Furthermore, Elle was unable to explain the phenomenon verbally, as she was able to do when dealing with time dilation. Therefore, it was not possible to identify evidence of Elle's conceptual understanding of the phenomenon. Additionally, without a verbal explanation, it was also not possible to identify any consistent mental representations, even in propositional format.

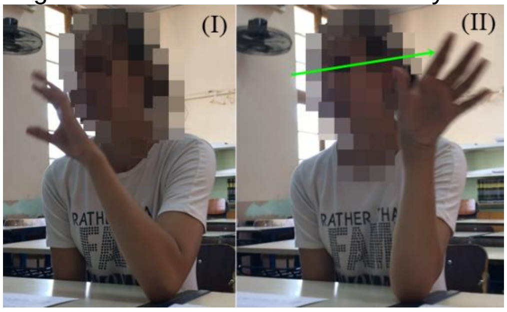
Iris had similar results, when explaining her answer to question 3, she stated that "when at a higher speed, it [the road] seems to be smaller". More explicitly, Iris also reported "I think it's just perception [that changes]". Therefore, she did not recognize the validity of the space contraction phenomenon.

Similarly to Elle, Iris seemed to sustain classical ideas of space, relating the difference in measurements to a different perception of an absolute space, due to her inability to observe and measure something moving at high speeds. In other studies, it has been observed that many students tend to interpret space contraction as a result of an error in measurement (Velentzas; Halkia, 2012).

Regarding her mental representation for spatial contraction, like Elle, Iris did not demonstrate any defined pictorial representation. Some indicators can be identified in her discourse. When discussing the activity with the simulation "Space Contraction", Iris stated her inability to "see" the simulation train moving at extremely high speeds, imagining "a blur [#1BOR 21:31]".

Additionally, the gesture #1BOR (Figure 6.7) performed by the student, where she moves her right hand quickly, is quite similar to the gesture #1MED performed by Elle, also indicating an undefined mental image.

Figure 6.7: Gesture #1BOR made by Iris.



Source: Author (2024).

It is likely that Iris also used representations acquired through daily experiences, such as seeing something moving quickly – psychophysical mediation, which cannot represent this relativistic phenomenon. She also was unable to explain the phenomenon verbally, indicating the absence of consistent mental representations in any format and lack of conceptual understanding of the phenomenon.

6.3 LIMITED UNDERSTANDING AND INCONSISTENT MENTAL REPRESENTATIONS

All students in this group demonstrated low levels of understanding for both time dilation and space contraction. Two representative students from this group are presented here. Some students, such as Eve, responded correctly to some questions, but were unable to provide a correct explanation of the phenomenon. Others, such as Nora, did not respond correctly to the questions and demonstrated a limited understanding of even Galilean Relativity.

6.3.1 Time Dilation

These students demonstrated similar ideas regarding time dilation. Discussing the activity with the “Time Dilation” simulation, which compared clocks inside a moving train and at the train station, according to an observer standing on the ground, it was possible to identify their conceptions.

For example, Eve stated:

> [...] the time at the station passes faster, because, the train is moving very fast, so, whoever is inside the train cannot understanding it [time passage], because it’s [the train] going too fast. But someone at the station is in normal time.

This statement suggests that Eve considered time dilation as a phenomenon based on perception. Although she correctly predicted the time on each clock, inside the train and at the station, she related the difference to the perception of time passage, rather than the time being measured differently.

Moreover, she also mentioned that someone at the station is “in normal time”, indicating an idea of absolute time and a privileged reference frame. Eve’s statements, relating time dilation to perception, aligned with an incorrect explanation of the phenomenon, indicate a lack of conceptual understanding of the student about this phenomenon. More studies have already reported tendencies for students to associate time dilation with a perceptual phenomenon (Dimitriadi; Halkia, 2012; Selçuk, 2011; Velentzas; Halkia, 2012).

During her explanation, Eve did not report “imagining” something and did not perform any depictive gestures, presenting no indication of the use of pictorial mental representations. The student also failed to explain correctly the phenomenon verbally, possibly indicating a lack of clear mental representations for time dilation in any format, pictorial or propositional.

Discussing the same activity, another student, Nora, stated:

> It [the train] is moving and the person [inside the train] wouldn’t realize that time was passing. To her, time would pass quicker than for someone outside seeing it [the train] passing. It’s like when we’re standing still doing something, time doesn't pass quickly for us, or when we are in a class we don’t like, time doesn’t pass quick, but if we are in a class we like, it seems that the time passes quicker.

According to her response, Nora related the phenomenon to a psychological perception of time passage within an absolute time frame. She did not consider that, from the station’s reference point, time would pass more slowly inside the train. Nora even compared this situation to everyday moments when time seems to pass more quickly due to engagement in an activity. Therefore, Nora presented evidence of a lack of conceptual understanding of the phenomenon of time dilation.

Furthermore, Nora did not present indicators of having mental representations (whether pictorial or propositional) for time dilation: she did not explain the phenomenon verbally, did not report “imagining” something, nor performed depictive gestures. Possibly, Nora was unable to comprehend the idea that time intervals between events are different when measured from different reference frames.

This difficulty likely arises from the fact that this phenomenon is not observed in everyday situations (Yavaş; Kızılcık, 2016), challenging the development of mental representations for it, even with the aid of external resources such as computer simulations.

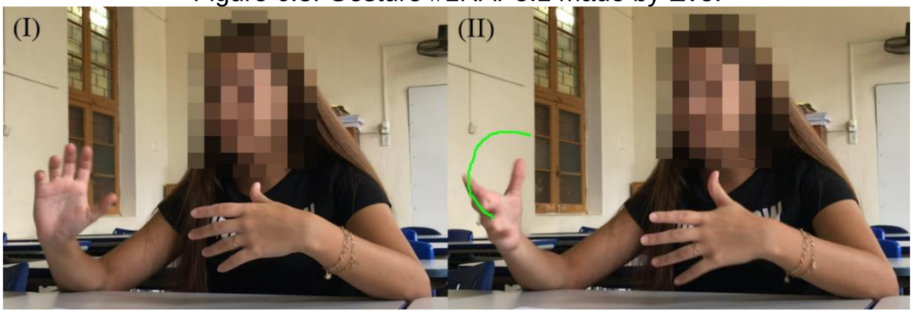
6.3.2 Space Contraction

Regarding the space contraction, Eve and Nora presented some differences, but both were unable to explain the phenomenon correctly. Eve recognized the difference in measurements of space for different reference frames but attributed it exclusively to perceptual effects. Discussing question 3 of the test, regarding the distance measured by a moving truck, she answered correctly “less than 100 m”:

> [...] when the speed is greater, you cannot have, see right, what is the distance. So, I imagined it would be less than 100 [meters] because it would be at an extremely high speed, and I wouldn’t have a way to measure the distance value.

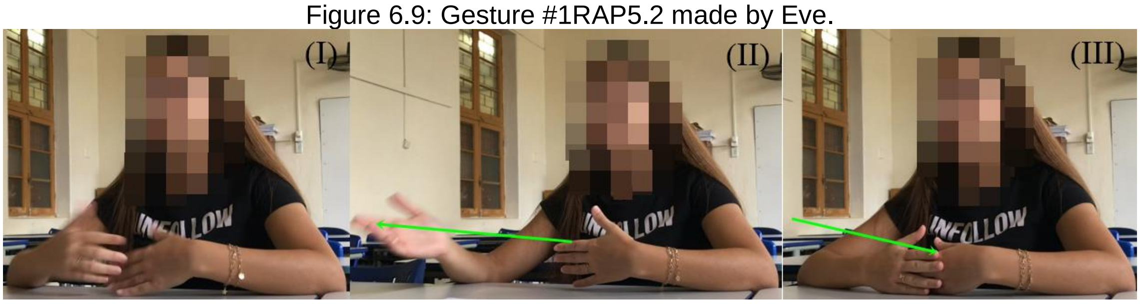
This statement by Eve indicates an idea of absolute space, relating changes to an inability to make correct measurements. Other indicators of this conception are the gestures performed by her. In gesture #1RAP5.1 (Figure 6.8), Eve moves her right hand rotating, indicating the idea of the impossibility of making the measurement. In gesture #1RAP5.2 (Figure 6.9), where the student moves her right hand quickly, indicating the movement of the truck, it indicates the absence of a clear mental image.

Figure 6.8: Gesture #1RAP5.1 made by Eve.



Source: Author (2024).

Figure 6.9: Gesture #1RAP5.2 made by Eve.



Source: Author (2024).

By her statement, Eve interpreted the space contraction as an apparent effect, a common misconception among students (Velentzas; Halkia, 2012). Moreover, she stated that observers at rest relative to the ground could not measure the correct size of the truck due to its “extremely high speed”. This answer, aligned with the gestures made by Eve, suggests that she did not have a clear mental image of the space contraction.

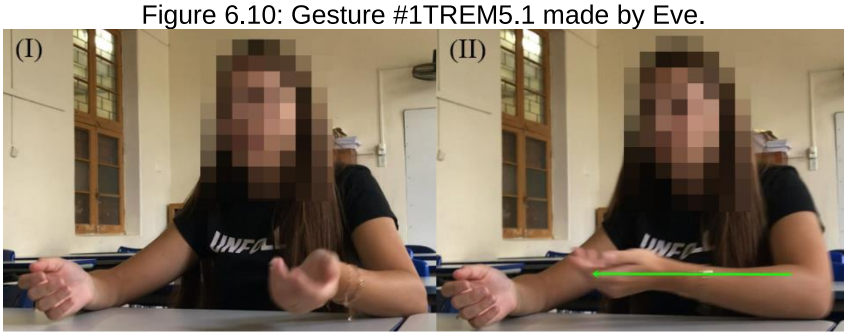
In the excerpt above, it is possible to observe an association of these differences only at “extremely high speeds”. This becomes more evident when Eve was questioned about a change in speed to 0.5c, and she stated: “it’s also an extremely high speed”; suggesting that she did not understand that the difference in measurements increases as the truck’s velocity increases.

This observation implies that Eve did not understand the role of the gamma factor and its connection to relative velocity, which increases exponentially; even though the space contraction is imperceptible in speeds observed in everyday life, it exists. This association by students of relativistic phenomena to only “extremely high speeds” has been observed in other studies (Gousopoulos; Kapotis; Kalkanis, 2016).

More evidence that Eve interpreted the phenomenon as apparent was identified discussing the activity with the “Space Contraction” simulation, which compared the length of a moving train measured by someone inside it and someone at the station. Eve presented an idea of absolute space, with only perceptual changes:

> [...] whoever is at the station wouldn’t be able to see the train correctly. Because it [train] would be going very fast [#1TREM5.1 22:59], then it would pass [#1TREM5.2 23:00] and they wouldn’t be able to measure it [the train]. But whoever is inside wouldn’t notice it [the change], because for someone inside it’s like being stationary.

Figure 6.10: Gesture #1TREM5.1 made by Eve.



Source: Author (2024).

Eve seemed to associate the differences in the measured length for the two reference frames to a perception of an absolute space or a measurement error, not that the measurement made is correct (Velentzas; Halkia, 2012). Moreover, Eve mentioned not being able to “see the train”, similarly to what she mentioned when discussing question 3 for the truck, and thus it seemed smaller, indicating the absence of clear mental simulations or mental images. The gestures #1TREM5.1 (Figure 6.10) and #1TREM5.2 (Figure 6.11), both in which the student moves her hand quickly, indicate the rapid movement of the train that cannot be correctly observed.

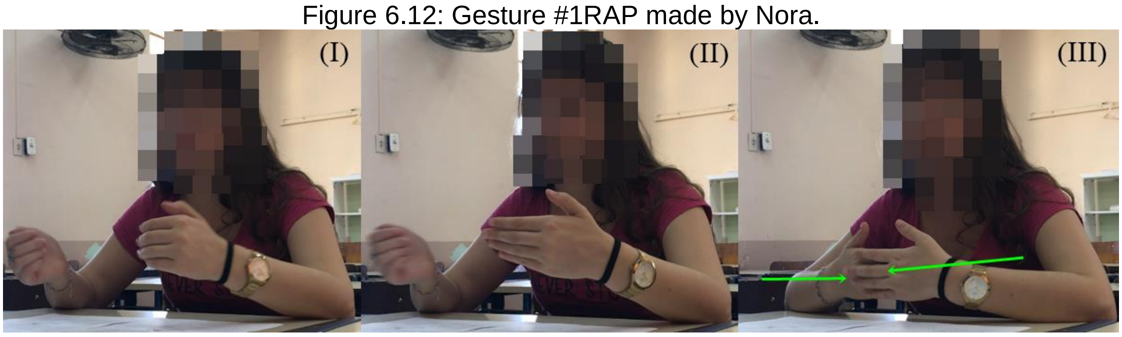
Figure 6.11: Gesture #1TREM5.2 made by Eve.

Source: Author (2024).

Furthermore, as Eve also could not explain the phenomenon verbally, she did not present indicators of propositional mental representations. With no defined mental representation, neither imagistic nor propositional, Eve was unable to imagine or explain these situations. This absence of solid mental representations became an obstacle to her acquire a complete conceptual understanding of Special Relativity.

Nora presented even greater difficulties regarding the space contraction. Unlike Eve, she did not recognize any effects at all. For question 3 of the test, she responded, “equal to 100 m” and stated: “it [truck 1] will only get to truck 2 faster, but it will have to travel the same 100 m; nothing will change the 100 meters”. This statement reveals an understanding of absolute space that cannot be altered for all observers.

Figure 6.12: Gesture #1RAP made by Nora.



Source: Author (2024).

When questioned about her visualization of this scenario, Nora answered: “I cannot imagine something so fast [#1RAP 16:28]” and performed the gesture #1RAP

(Figure 6.12), where she positioned her hands closed in front of her chest, then opened them and moved one quickly towards the other to symbolize rapid movement.

The gesture and Nora's report indicate that she did not develop a clear mental image of the truck moving at 0.7c. According to her description, Nora generated an undefined mental image of something moving at high speeds. As presented earlier, this undefined mental image can be associated with daily experiences through psychophysical mediation, such as observing a vehicle moving at high speeds (Figure 6.6). This difficulty in match relativistic effects with daily experiences ultimately becomes an obstacle to understanding the phenomenon (Yavaş; Kızılcık, 2016).

Moreover, Nora also could not explain the phenomenon of space contraction verbally. Therefore, it was not possible to identify evidence of the use of any format of mental representations, whether pictorial or propositional. Even after interacting with resources that can represent the phenomenon through hypercultural mediation, which conflicts with her daily experiences, Nora still tried to explain the situation with classical concepts learned previously. Consequently, she was unable to develop a conceptual understanding of the phenomenon.

7 RESULTS COHORT 2

Similar to Cohort 1 students, the eight students from Cohort 2 were divided into three groups. The results are presented in this chapter according to these groups: satisfactory understanding and consistent mental representations (3); partial understanding and developing mental representations (3); and limited understanding and inconsistent mental representations (2).

The following section discusses the cases of four exemplary students, half of the sample, with two exhibiting satisfactory understanding, one exhibiting partial understanding, and one exhibiting limited understanding. Both the conceptions and representations of these students were analysed in relation to the main concepts within the General Relativity Theory, including curved spacetime and gravitational time dilation.

Given that the foundation for understanding General Relativity lies in the concept of a four-dimensional curved spacetime (Kersting, 2019b; Mcinerney; Sutton, 2024), which is quite abstract and non-intuitive, a greater focus was placed on analysing the conceptions and representations of curved spacetime. This analysis primarily led to the categorization of students into the three groups.

7.1 SATISFACTORY UNDERSTANDING AND CONSISTENT MENTAL REPRESENTATIONS

The students categorized in this group demonstrated a good understanding of curved spacetime, using this understanding to explain various phenomena. Therefore, these students were able to correctly answer many questions on the test and provide coherent explanations for their answers during the interviews. Students Maia and Nina showed a good understanding of the rubber-sheet analogy, presenting evidence of pictorial mental representations of curved spacetime.

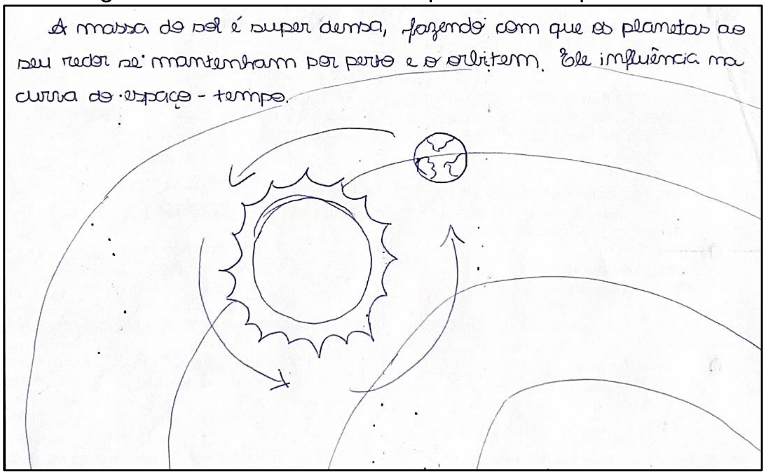
7.1.1 Curved Spacetime

The question 9[8] on the test asked: “Explain, as you were telling to a classmate, why the Earth moves in an orbit around the Sun. To do that, you can use text, drawings, graphs, diagrams etc”. To answer this question, Maia mentioned the influence of the

[8] The test presented in Appendix A is the last version of it where this question was moved to the final of the test (question 11).

Sun’s mass on the curvature (Figure 7.1): “The Sun’s mass is super dense, making the planets around it to remain close and orbit it. It [Sun] influences the curve of spacetime”.

Figure 7.1: Maia’s answer to question 9 of post-test.

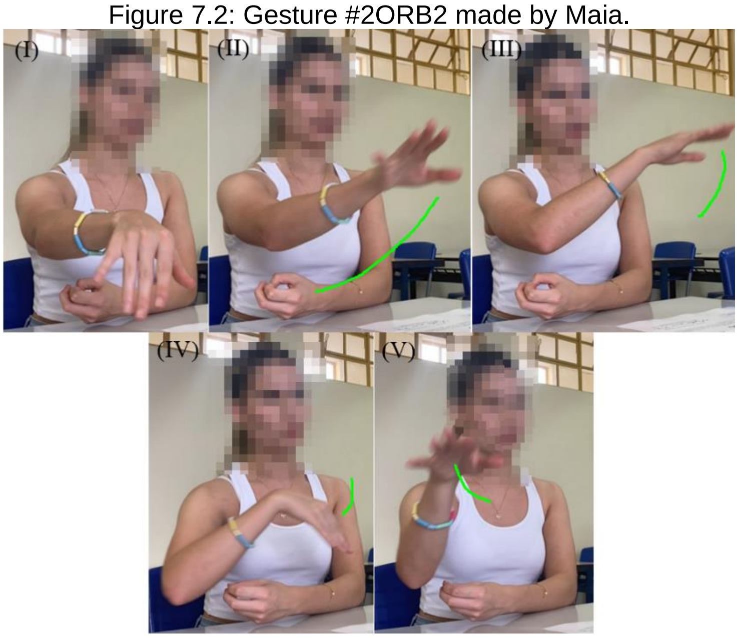


Source: Author (2024).

Maia drew a diagram of the Earth’s orbit and attempted to represent the “curve of spacetime” through the lines at the back of the diagram. As the student explained in the interview:

> I put the diagram of the spacetime curves here. Because I know that according to Einstein’s theory, the idea is that planets, the things follow the path [#2ORB2 16:54] of the spacetime curve. And also that everything with mass, density, ends up forming a hole in spacetime, so this attracts [#2GRAV 17:05] things around it. That’s what I thought of the Sun, the Sun has a mass that’s not as large as a black hole, but it’s a sufficient mass to form a hole like that, to make some kind of [#2GRAV2 17:16] influence on the curve.

Figure 7.2: Gesture #2ORB2 made by Maia.



Source: Author (2024).

Figure 7.3: Gestures #2GRAV and #2GRAV2 made by Maia.

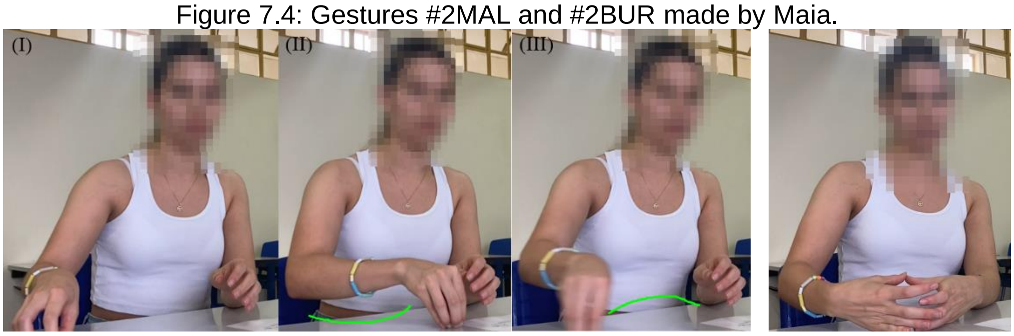
Source: Author (2024).

When discussing about the “curve of spacetime”, Maia performed the gesture #2ORB2 (Figure 7.2), where she moves her right hand in a counterclockwise circle, changing the height of her hand, possibly indicating the curvature. Then, Maia mentions a “hole” in spacetime caused by massive objects and performs the depictive gestures #2GRAV (Figure 7.3 left), where she moves her right hand horizontally, indicating attraction, and #2GRAV2 (Figure 7.3 right), where she moves her right hand down, indicating the “hole” caused by the Sun’s mass.

These dynamic gestures, aligned with Maia’s discourse, indicate that she has developed a mental simulation for Earth’s orbit and connected it to the idea of the “curve of spacetime”. As Maia mentioned a “hole”, moving her hand down, this is an indication that she has associated this curvature caused by massive objects with “downwards”. This is one of the limitations of the rubber-sheet model presented in other studies (Postiglione; De Angelis, 2021b), which the student mentioned remembering.

> I remember that experience with the marbles that you showed us, with the blanket, the sheet [#2MAL 17:38], right. Then you throw the marbles and then it forms, kind of a gap [#2MAL 17:44], like that, forms a little hole [#2BUR 17:45], and then it attracts [#2MAL 17:47] the marbles around it too.

Figure 7.4: Gestures #2MAL and #2BUR made by Maia.



Source: Author (2024).

When talking about the activity with the physical rubber-sheet model, Maia mentioned a “little hole” performing the gesture #2BUR (Figure 7.4 right), where she placed her two hands indicating a sphere. The student also mentioned the movement of the marbles, performing the gesture #3MAL (Figure 7.4 left), moving her right hand in a counterclockwise circle, probably comparing the movement of the marbles with the movement of the planets.

Since Maia mentioned the physical rubber-sheet model directly and, as indicated by her depictive gestures, similar to those performed earlier, it is likely that Maia developed her mental representation through the interaction with this psychophysical resource. Following the interview, Maia demonstrated that she also used other external resources to develop her mental representation.

> I remember that you showed us a website, a simulation, that you were approaching [#2DIS7 19:25], with a ship, I think, and some points [#2PON 19:31] that you were moving and showing the difference in time and the difference in the distortion [#2DIS8 19:36], of distances. I remember this. [...] I remember that it was on the projector, the points that were changing, there was a little red dot, a little green dot, a little blue dot [#2PON2 19:48] ... And then each dot was changing [time].

Figure 7.5: Gestures #2DIS7 and #2PON made by Maia.

Source: Author (2024).

Figure 7.6: Gesture #2DIS8 made by Maia.

Source: Author (2024).

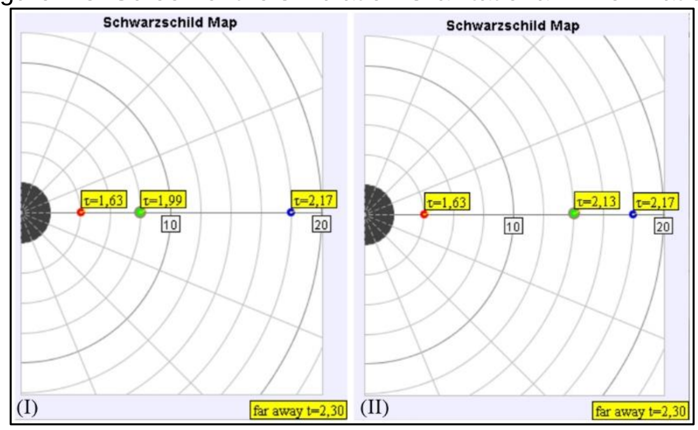
According to her description, Maia also associated her ideas about the “curve of spacetime” with the simulation “Gravitational Time Dilation” used in one of the classes through the projector (Figure 7.8). She mentioned the proximity of the massive object in the simulation, performing the gesture #2DIS7 (Figure 7.5 left) where she moves her right hand over the table, and related it to the distortion caused by it.

Figure 7.7: Gesture #2PON2 made by Maia.

Source: Author (2024).

Maia mentioned the different points indicated in the simulation, performing the gesture #2PON2 (Figure 7.7), where she uses her right index finger to mark three points, and the difference in distortion presented between them, performing the gesture #2DIS8 (Figure 7.6), where she moves her right hand to the right over the table. As indicated by Maia's discourse, the "distortions" presented by the simulation helped her understand the idea of spacetime curvature.

Figure 7.8: Screen of the simulation Gravitational Time Dilation.

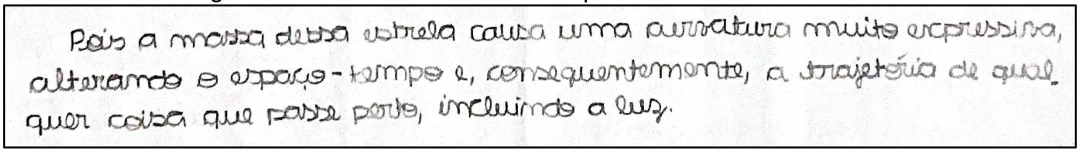


Source: Author (2024).

In this way, it is possible to observe the influence of a hypercultural external resource on the construction of Maia's mental representation. The physical rubber-sheet model, associated with the computer simulation, helped Maia to develop *drivers* in the form of a pictorial representation of spacetime curvature. Furthermore, Maia demonstrated using this mental representation to solve question 9 of the post-test about the Earth's orbit.

Maia also demonstrated she has used her mental representation of spacetime curvature to answer an exercise solved in class. The question asked: "Consider the star Sirius, with a mass more than twice the Sun's mass. Why would objects near it describe curved trajectories? Would a light beam passing nearby be curved?". Maia answered: "Because the mass of that star causes a very expressive curvature, changing spacetime and, consequently, the trajectory of anything that passes nearby, including light" (Figure 7.9).

Figure 7.9: Maia's answer to the question solved in classes.

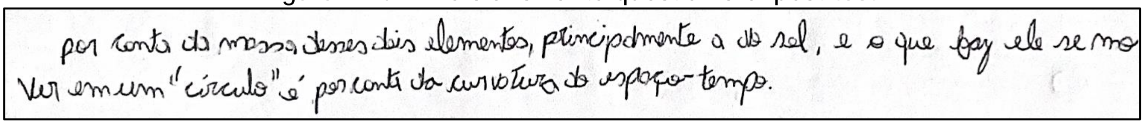

Pois a massa dessa estrela causa uma curvatura muito expressiva, alterando o espaço-tempo e, consequentemente, a trajetória de qualquer coisa que passe perto, incluindo a luz.

Source: Author (2024).

In her response, Maia mentioned the curvature caused by mass and the alterations caused by it in spacetime. Therefore, this is an indication that she accessed the same mental representation she used in the previous situation to solve a problem involving the light bending. As indicated by the Maia's results, this mental representation was constructed through interaction with psychophysical and hypercultural mediations. The use of different external resources by Maia to construct a coherent mental representation highlights the importance of complementing the rubber-sheet model with other resources, as emphasized in other studies (Kersting; Steier, 2018).

Nina also answered question 9 of the post-test with similar conceptions to Maia, stating that the Earth's orbit is caused due to "the mass of these two elements, mainly the Sun's mass, and what makes it [Earth] move in a 'circle' is the curvature of spacetime" (Figure 7.10).

Figure 7.10: Nina's answer to question 9 of post-test.


por conta da massa desses dois elementos, principalmente a do sol, e o que faz ele se mover em um "círculo" é por conta da curvatura do espaço-tempo.

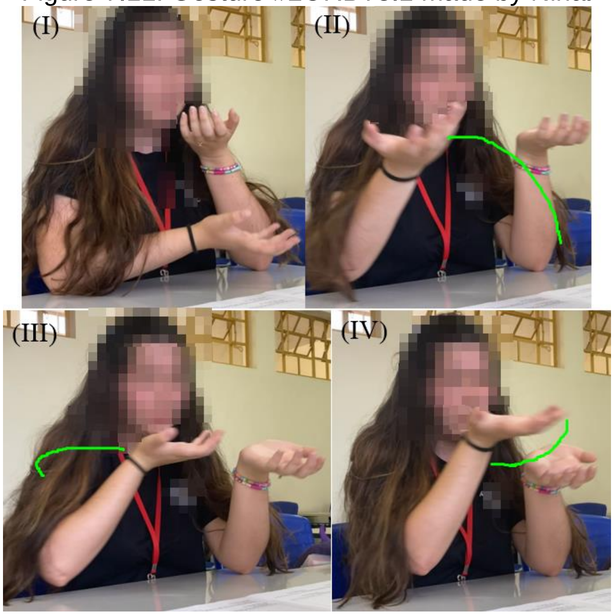
Source: Author (2024).

It is possible to observe from Nina's response that she associated the Sun's mass with the curvature of spacetime. This curvature would have as a consequence the Earth's orbit, which would be "forced" to follow this path.

> I just remembered that the Sun is a massive object... I don't know how I came to that conclusion. But I imagined that, due to the curvature, the Earth would be forced to make that movement [#2ORBT3.1 23:49 2x].

According to Nina's explanation, as the Earth is moving in a curved spacetime, this would cause its orbit. The student performed the depictive gesture #2ORBT3.1 (Figure 7.11) during her explanation, where she positioned her left hand, possibly representing the Sun, and then moved her right hand in a circular motion, indicating the Earth's movement. Nina changed her hand's height during the movement, possibly connecting it to the curvature.

Figure 7.11: Gesture #2ORBT3.1 made by Nina.

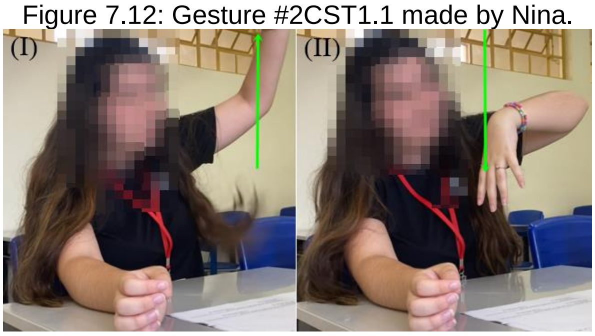


Source: Author (2024).

Nina's explanation, along with her dynamic gesture, are indicators that she developed a pictorial mental representation of spacetime curvature. Nina used this mental representation to explain the Earth's orbit. She then provided indications of the resources that helped her to develop her mental representation.

> Regarding the curvature, I remembered those situations we did with Einstein. He was up there on the tower and then he jumped [#2CST1.1 24:35] and then for the person outside [#2CST1.2 24:38] he was jumping straight, but in reality, he would be jumping like this [#2CST1.3 24:42]. [...] when I think about this curvature, it's easier to remember the image. [The curvature] makes it [Earth] move like this [#2ORBT3.2 25:28 3x].

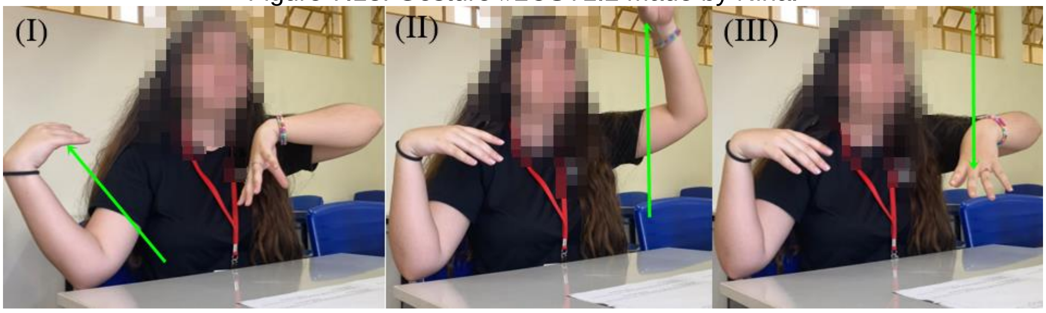
Figure 7.12: Gesture #2CST1.1 made by Nina.



Source: Author (2024).

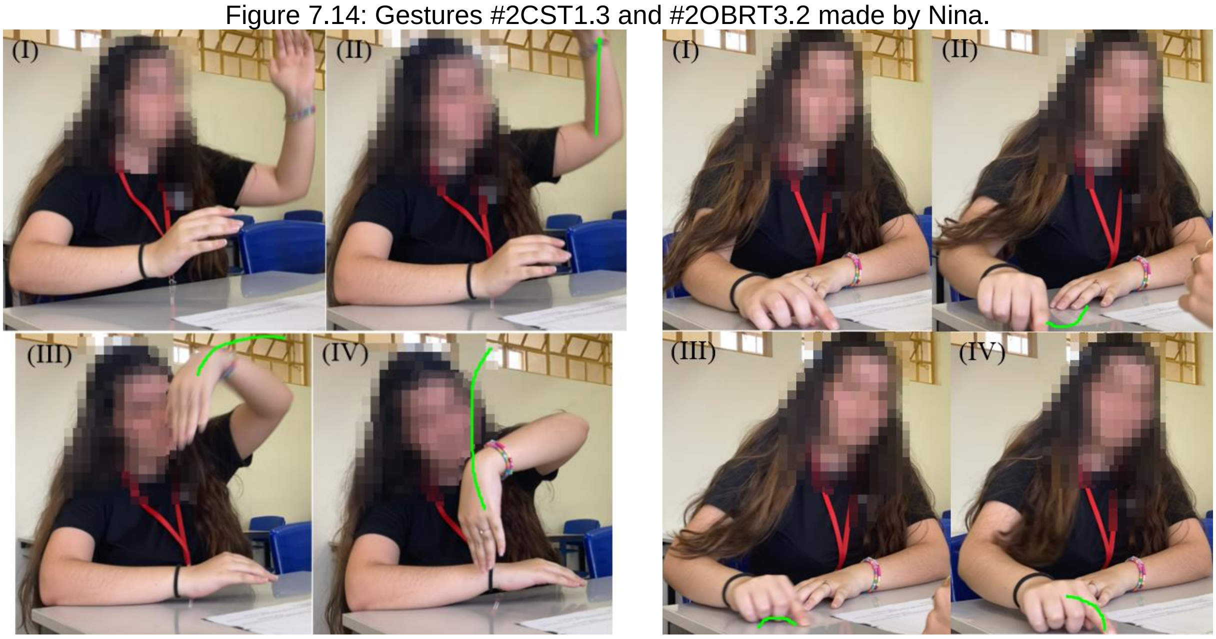
According to Nina's report, she constructed her mental representation of curvature through an activity with a hypercultural resource, the "Warped Time Model". This model demonstrates free-fall through Einstein's perspective, depicting curvature in the temporal dimension. Nina stated that she remembered how the timeline was curved, performing the gesture #2CST1.3 (Figure 7.14 left), where she moves her right hand in a curved descending motion.

Figure 7.13: Gesture #2CST1.2 made by Nina.



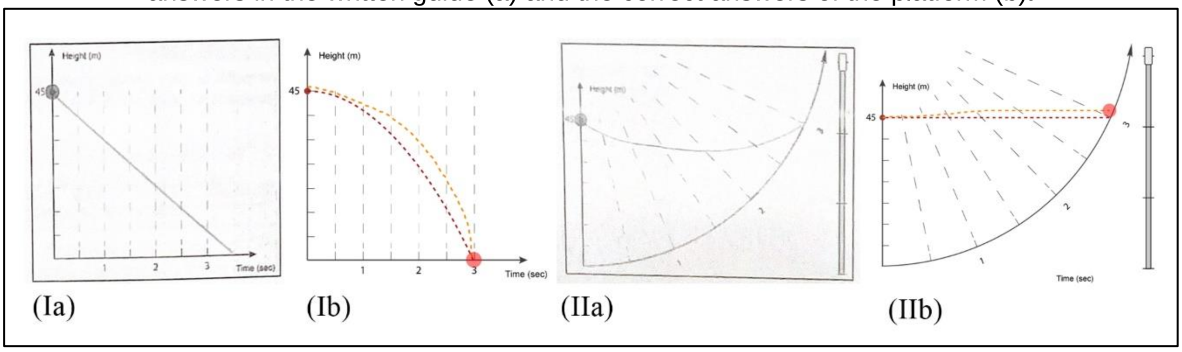
Source: Author (2024).

Figure 7.14: Gestures #2CST1.3 and #2OBRT3.2 made by Nina.



Source: Author (2024).

Figure 7.15: Free-fall diagrams in Newtonian (I) and Einsteinian (II) perspectives, with the Nina's answers in the written guide (a) and the correct answers of the platform (b).



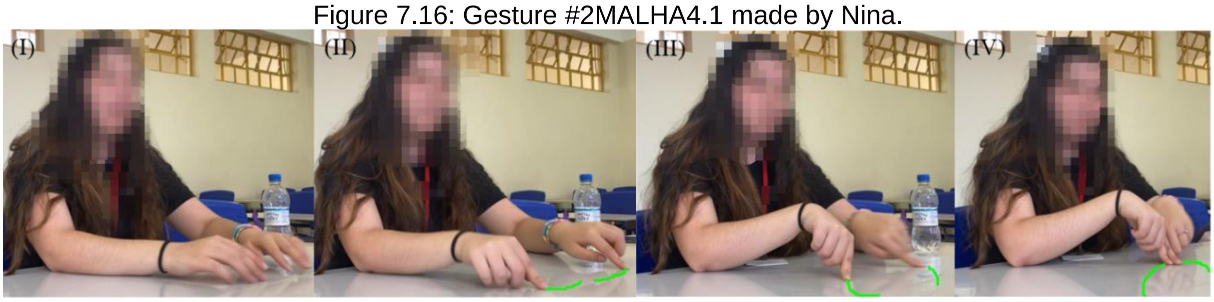
Source: Author (2024).

In the activity with this resource, students were supposed to draw the trajectory of free-fall in diagrams of height over time, for the Newtonian and Einsteinian perspectives, which presented the diagram with curved time (Figure 7.15). The student related this curvature to the cause of the Earth's movement around the Sun, performing the gesture #2ORBT3.2 (Figure 7.14 right), in a circular motion with her right index finger, indicating the Earth's orbit.

Through Nina's description and gestures, it is possible to observe the influence of the use of this hypercultural resource on the development of her mental representation of curvature. Nina also stated that she remembered the physical rubber-sheet model used in class:

> I liked the activity with the sheet [#2MALHA4.1 34:37] that we did, where we were throwing the marbles, and then it was [#2MALHA4.2 34:42] getting deep down underneath [#2MALHA4.3 34:43].

Figure 7.16: Gesture #2MALHA4.1 made by Nina.



Source: Author (2024).

Figure 7.17: Gestures #2MALHA4.2 and #2MALHA4.3 made by Nina.

Source: Author (2024).

When Nina mentioned the activity with the rubber-sheet model, she performed the gesture #2MALHA4.1 (Figure 7.16), moving her two index fingers over the table forming a circle. Possibly this gesture indicates the movement performed by the marbles used in the activity. Additionally, this gesture is similar to the gesture #2ORBT3.2 performed earlier by her to indicate the Earth's orbit. Then, Nina performed the gestures #2MALHA4.2 and #2MALHA4.3 (Figure 7.17), indicating the deformation caused by the marbles on the sheet.

Observing Nina's discourse and her depictive gestures, it is possible that she related what was observed in the psychophysical resource, physical rubber-sheet model, with the explanation for the Earth's movement around the Sun. Therefore, similarly to Maia, Nina probably developed her mental representation of curvature from the interaction with psychophysical (physical mesh model) and hypercultural (Warped Time Model) mediations. By combining more than one level of external mediation, Nina was able to develop a pictorial mental representation and use it to answer question 9 of the test.

7.1.2 Gravitational Time Dilation

Maia also demonstrated a good understanding of gravitational time dilation. Her conceptions were observed when the student explained her response to question 7 of the post-test: “Two astronauts are in the International Space Station. Consider that one of them goes down to Earth, do an activity and finished it in 8 h, according to his watch. What was the time interval for the astronaut who stayed in the space station?”. Maia stated that less time would pass for the astronaut who waited in the space station:

> I remembered the movie we discussed, the spaceship... How can I explain... The mass of the planet changed how time passed, something like that... And then I started thinking about it, if he [astronaut] goes down and is closer to that mass, then time would pass more slowly [for him] [...] I thought there was a difference due to the planet’s mass... And, also thinking about the movie [scene], which had a time difference.

It was possible to observe in her explanation that Maia correctly related the time passage being slowed down due to the proximity to a massive. The student claimed to remember a scene from the movie *Interstellar*, which was presented in class, where the characters go to a planet near a black hole and have their “watches” slowed down. Maia correctly transposed the situation from the movie to the post-test question.

The student also responded correctly to question 8: “Consider now that the astronauts are on a spaceship close to a supermassive black hole. One of them goes in a mission and gets closer to the black hole, finishing the mission in 3 h, according to his watch. What was the time interval for the astronaut who stayed on the ship?”. Maia stated that much more than 3 h would pass for the astronaut who waited on the ship:

> Because near the black hole, the distance, the differences, everything gets amplified. So even if he [astronaut] had little time near the black hole, that’s a lot of time, then the difference is very striking.

Subsequently, Maia was questioned about a situation involving some object such as the Sun, with less mass than the black hole but more mass than the Earth.

From Maia's explanation, it was possible to observe that she correctly related the amount of mass to the influence caused on the passage of time:

> I think there would be a difference, but not that great. I think there was a situation on the ENEM that they talked about something about this. But I think it wouldn't be a difference that striking. [...] The Sun, although it's very large for us, in our Solar System, it's nothing compared to a black hole.

Therefore, analysing Maia's responses and explanations to questions 7 and 8 of the post-test, it is possible to infer that she developed a good understanding of gravitational time dilation. Since Maia did not report any imagery processes while explaining her responses, nor did she perform depictive gestures, probably she developed a propositional mental representation of time dilation. In this representation, Maia correctly established a relationship between the time passage and massive objects.

Furthermore, it is possible to observe indications that Maia developed her mental representation from the interaction with cultural mediation. As the student mentioned earlier, when responding to the questions, Maia remembered a scene from the movie *Interstellar*. Additionally, at another point, she also mentioned the first photos of black holes presented in class:

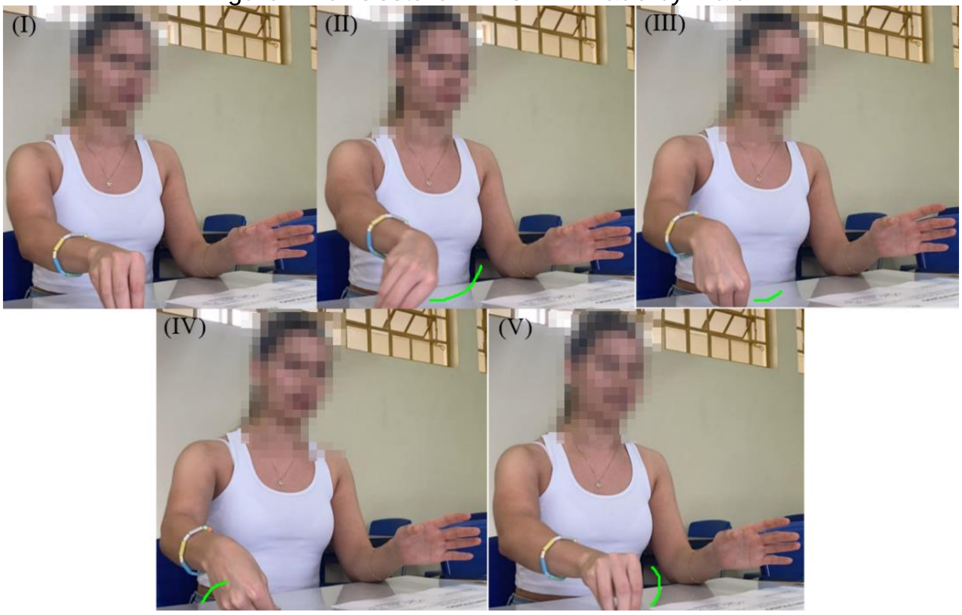
> Some images were shown, like captures, of the black hole, and how we see a light around it [#2HOLE3 15:15], but it [black hole] doesn't have light, because that's just [#2HOLE4 15:20] the action of the black hole in the image.

Figure 7.18: Gesture #2HOLE3 made by Maia.

Source: Author (2024).

Therefore, probably Maia used both the movie scene and the images seen in class to develop her propositional mental representation of time dilation. While describing the images, Maia performed the gesture #2HOLE3 (Figure 7.18), where she placed her right hand indicating the black hole, and the gesture #2HOLE4 (Figure 7.19), where she moved her right hand in a circle, indicating the surroundings of the black hole. However, these gestures indicate that she has an mental image for the black hole, but not for the phenomenon of time dilation itself.

Figure 7.19: Gesture #2HOLE4 made by Maia.



Source: Author (2024).

Nina, on the other hand, did not present a solid understanding of gravitational time dilation. For question 7, Nina presented the idea that “time is different in space” and, therefore, would pass less time for the astronaut in the space station:

> I imagined it would be a little less than 8 h, due to the time difference there [space station]. I think it’s because of the mass of the objects around it. And then, as one of the astronauts comes to Earth, in our ‘normal time’, in his clock passed 8 h, but for the astronaut who stayed there [space station], it passed less [than 8 h].

Although Nina mentioned “the mass of objects”, she did not seem to recognize the Earth as one of those objects. Nina even used the expression “normal time” to refer to the time passed for someone on Earth. However, when dealing with question 8 of the post-test, involving a black hole, she answered correctly:

> Here I imagined it in relation to mass. Because a black hole is a very massive object. And then, due to that, time passes very differently there [near the black hole] compared to here [spaceship]. So, for that person who stayed here [spaceship], I imagined it would pass much more time than 3 h.

Nina correctly responded that time would be slowed down near the black hole. The student also correctly related that the cause of the effects would be the mass of the black hole. Therefore, it is possible that Nina did not recognize the Earth as a massive object capable of influencing the time passage. This conception was more evident when Nina was questioned about what would be the massive object in question 7, as she had mentioned in her explanation that the mass caused the effect on time.

> Maybe [it would be] also 8 h. Because there's nothing [massive] there to interfere with the time, right? Like, he [astronaut] is outside in space, so maybe... I don't know, maybe I would mark also 8 h.

Observing Nina's discourse, it is possible to observe that she did not consider the Earth when reflecting on question 7. Nina even stated that she would change her answer to "also 8 h", meaning that there would be no difference between the time intervals for the astronaut who went down to Earth and for the one who waited in the space station. This difficulty of Nina in recognizing Earth as a massive object aligns with the difficulties observed with other secondary students to associate Einstein's ideas with their daily lives, even if they understand these ideas (Kersting; Henriksen; Bøe; Angell, 2018).

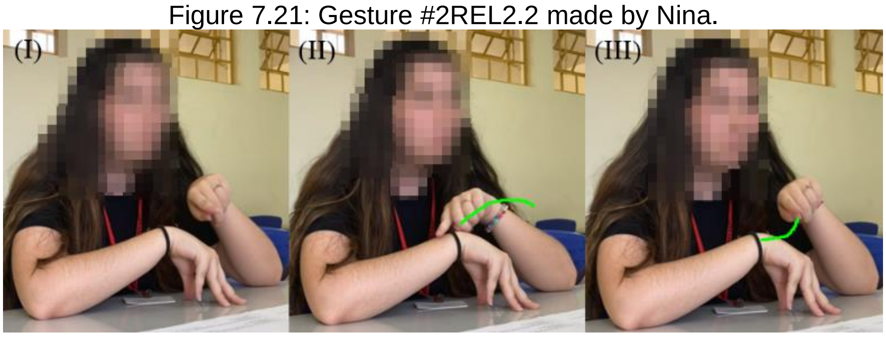
When questioned about whether she imagined something or remembered something to answer to the question, Nina mentioned a cultural resource, the movie *Interstellar*:

> I imagined the movie [Interstellar] too, which we watched, that when they [characters] went down to that planet, time passed much less for them than for the man who was on the spaceship. [...] I remember that her father gave her [Murphy] a watch. And then they both had to compare the time, on her watch and on his watch [#2REL2.1 30:41], to see how different it would be [#2REL2.2 30:44 2x].

Figure 7.20: Gesture #2REL2.1 made by Nina

Source: Author (2024).

Figure 7.21: Gesture #2REL2.2 made by Nina.



Source: Author (2024).

When explaining what she remembered, Nina mentioned the watches of the movie characters, performing the gesture #2REL2.1 (Figure 7.20), where she indicated a watch on each hand, and #2REL2.2 (Figure 7.21), where she moved her right index finger over her left hand, indicating the watch’s hand. Analysing these results, it is possible that Nina has a developing pictorial mental representation of time dilation, originating from this interaction with a cultural mediation.

Even without recognizing the Earth as a massive object, Nina demonstrated understanding of the functioning of gravitational time dilation. When questioned about what would influence the passage of time, Nina stated that both mass and distance from the object would influence it:

> I think it’s linked to both, the amount of mass and the distance. Because I remember we did an activity on the computer, I think, and there were three [#2PONT 21:41 3x] points. I think they had different colours, yellow, red, blue, something like that. And then, when we got closer [#2APR3 21:49] to the black hole, time passed slowly, but a small distance [#2DIF3 21:55] we take, that time already changed a lot. Because the closer to it the bigger the difference.

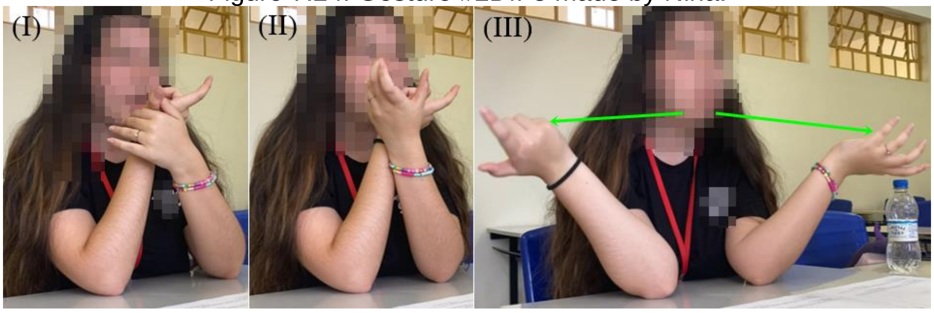
Figure 7.22: Gesture #2PONT made by Nina.

Source: Author (2024).

Figure 7.23: Gesture #2APR made by Nina.

Source: Author (2024).

Figure 7.24: Gesture #2DIF3 made by Nina.



Source: Author (2024).

Again, Nina used depictive gestures to express her ideas. Firstly, she performed the gesture #2PONT (Figure 7.22), indicating the three different points of the simulation with her right hand, and then the gesture #2APR (Figure 7.23), where she moves her right hand indicating the points closer to the black hole. Finally, Nina performed the gesture #2DIF3 (Figure 7.24), where she brought her two hands together, using her index finger and thumb to mark a measurement, and then separated the two hands moving them horizontally, one to each side. This gesture indicates the different points, at different distances from the black hole, with their different measurements of time.

According to her report, besides the cultural mediation mentioned earlier, Nina also used hypercultural mediation to develop her understanding of time dilation. Since the simulation “Gravitational Time Dilation” (Figure 7.8) mentioned by Nina was presented by the teacher on the projector, through discussions with the students, it is possible that social mediation also has had an influence on the development of Nina’s mental representation. Therefore, Nina used the interaction with different levels of mediation (cultural, hypercultural, and social) to construct *drivers* in the form of a pictorial mental representation that helped her understand the phenomenon.

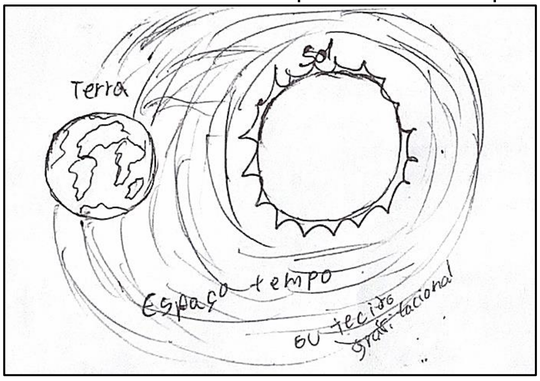
7.2 PARTIAL UNDERSTANDING AND DEVELOPING MENTAL REPRESENTATIONS

Students with partial understanding presented an initial comprehension of curved spacetime. Therefore, these students presented mental representations still in development about the phenomena studied. This group will be exemplified by the case of student Leia.

7.2.1 Curved Spacetime

In her answer to question 9 of the post-test, which dealt with the reason for the Earth’s orbit, Leia demonstrated an initial understanding of curved spacetime. The student mentioned the “gravitational fabric” and “spacetime’ but did not mention the mass of the Sun or the curvature caused by it (Figure 7.25).

Figure 7.25: Leia’s answer to question 9 of the post-test.



Source: Author (2024).

During the interview, Leia explained what she was trying to represent through her drawing on the post-test:

> Because the Earth isn’t... the whole universe, the planets [#2AST 17:56] aren’t just floating around in space [#2JOG 18:00]. There’s, the gravitational fabric, the general spacetime that makes it [Earth] [#2ORB 18:06] spin.

Figure 7.26: Gestures #2AST and #2JOG made by Leia.

Source: Author (2024).

Figure 7.27: Gesture #2ORB made by Leia.

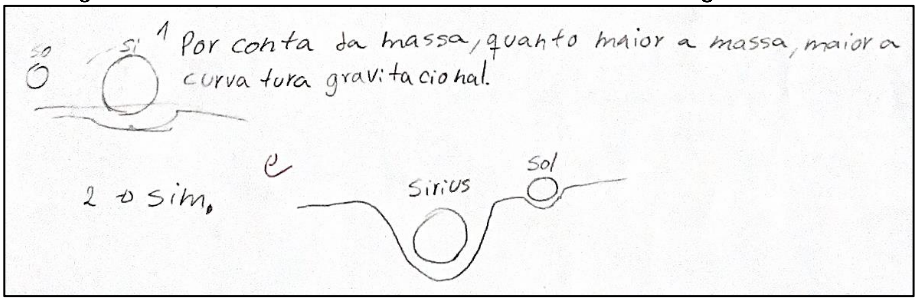
Source: Author (2024).

In her explanation, Leia asserted that the “gravitational fabric” of “spacetime” would be the cause for the Earth’s movement, as well as for the other planets, around the Sun. Leia performed the depictive gesture #2AST (Figure 7.26 left), positioning her right hand to represent celestial bodies, and the gesture #2JOG (Figure 7.26 right), moving her right hand as if throwing a ball, to indicate that these bodies would not be “loose” in the universe. Then, Leia performed the gesture #2ORB (Figure 7.27), indicating an orbit in a clockwise direction with her right index finger. This gesture indicates the movement caused by this "gravitational fabric".

Leia demonstrated some understanding of the cause of the Earth's orbit, as well as a mental representation of it, as indicated by her depictive gestures. However, she did not demonstrate an understanding of how this "gravitational fabric" influences the movement of stars, nor did she connect these ideas with the mass of objects.

However, observing her answer to an in-class exercise, Leia related mass to the curvature caused in this gravitational fabric. The question asked: "Consider the star Sirius, with a mass more than twice the Sun's mass. Why would objects near it describe curved trajectories? Would a light beam passing nearby be curved?". Leia responded: "Because of mass, the greater the mass, the greater the gravitational curvature. Yes." (Figure 7.28).

Figure 7.28: Leia's answer to the exercise solved during the classes.

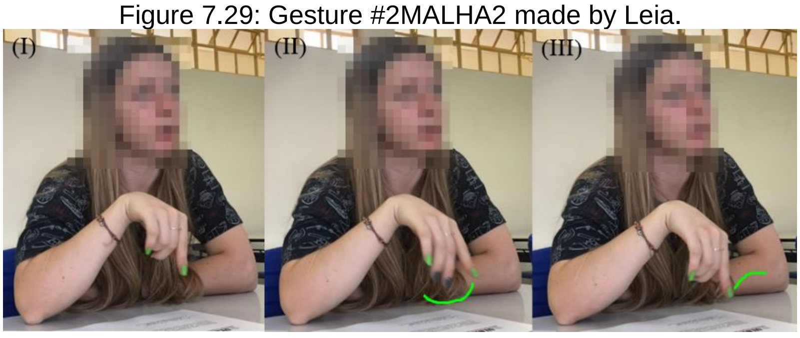


Source: Author (2024).

Analysing Leia's answer, both her text and her drawings indicate an idea of spacetime curvature. Possibly, the student is still developing her mental representation of this concept, which is not yet consolidated. When asked what she remembered when responding to question 9 of the post-test, Leia mentioned the activity with the physical model of the rubber-sheet analogy and the teacher's explanations.

> [I remember] the classes, of you talking in class. [...] I think it was the marbles [#2MALHA2 18:37], I think it was that. [...] that's why I put the gravitational fabric.

Figure 7.29: Gesture #2MALHA2 made by Leia.



Source: Author (2024).

By mentioning the activity with the marbles, Leia performed the depictive gesture #2MALHA2 (Figure 7.29), indicating an orbit in a clockwise direction with her right index finger. Since this gesture is similar to the gesture #2ORB, performed by the student when explaining her response, it is likely that Leia began to develop her pictorial mental representation from the interaction with the psychophysical resource of the rubber-sheet analogy.

7.2.2 Gravitational Time Dilation

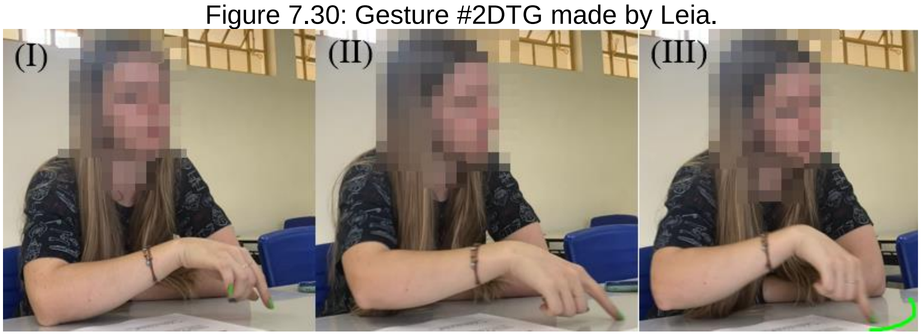
Leia presented an understanding still in development regarding gravitational time dilation. In question 7 of the post-test, where one astronaut goes down to Earth and the other waits in the space station, Leia responded that it would take “much more than 8 h” for the astronaut waiting. When explaining her answer, it was possible to observe that Leia established a relationship with the simulations used in the classes:

> Here I chose much more because I remember that, in the simulations you showed, the time interval was very different from [point] one to another. Therefore, I put much more than 8 h.

It is possible to observe that Leia transposed the situation seen in the simulation directly to the question, even though the simulation represented a black hole, and the question dealt with Earth. Possibly, Leia associated the same type of situation, but without recognizing the role of the Earth’s mass in the passage of time. Subsequently, Leia described a bit more about the simulation and even mentioned the black hole represented.

> I think there was a [simulator] that you showed, that when it got close, I don’t remember what it was, I think it was a black hole... I think there were lines [#2DTG 14:25] and the farther away [#2DTG2 14:27] it passed [more]. […] This one [gravitational time dilation simulation].

Figure 7.30: Gesture #2DTG made by Leia.

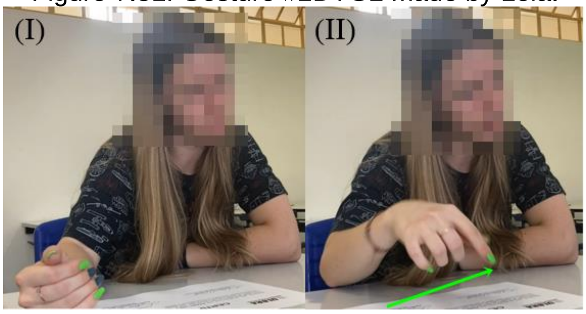


Source: Author (2024).

While describing the simulation she remembered, Leia mentioned the different distances and different time passages. She even performed the gesture #2DTG

(Figure 7.30), where she used her right index finger to represent the different lines of the simulation, and the gesture #2DTG2 (Figure 7.31), where she moved her right hand towards her body, indicating the different distances. Therefore, it is possible to observe that Leia only associated the distance between the two astronauts to answer the question, without considering the mass of the Earth.

Figure 7.31: Gesture #2DTG2 made by Leia.



Source: Author (2024).

This strong association with distance can also be observed in her explanation for question 8, regarding time dilation involving a black hole:

> This [question] was the same situation from that simulation. Because I think the spaceship wasn't in such a great distortion for he [astronaut] to goes down [#2DES 15:41] to make such difference. I think it was a bit more, and not much.

Figure 7.32: Gesture #2DES made by Leia.

Source: Author (2024).

It is possible to observe that Leia considered the distance between the two astronauts, the one that approaches the black hole and the one who waits in the ship, to be small, and therefore the time difference would also be small. During her explanation, the student performed the gesture #2DES (Figure 7.32), moving her right hand downwards, indicating the distance between the two astronauts. This gesture, like the ones performed earlier, indicates that Leia possesses a developing pictorial mental representation, about the different distances from the black hole and the different time passages.

Following the interview, Leia recognized that the black hole causes more effects on the spacetime around it than Earth, and connected this to the black hole's mass:

> The time and the distance around the black hole [#2HOR 20:12] are different from what they would be on Earth. [...] As you get closer to the black hole, it starts to suck you in [#2HOLE2 22:26], and time will pass differently. […] The mass, it [black hole] has much more mass, is that.

Figure 7.33: Gesture #2HOR made by Leia.

Source: Author (2024).

Figure 7.34: Gesture #2HOLE2 made by Leia.

Source: Author (2024).

When she mentioned the effects caused around the black hole, Leia performed the gesture #2HOR (Figure 7.33), moving her right hand in a counterclockwise direction. This indicates that the student possesses a mental image for the black hole itself. When explaining her ideas, Leia also presented an idea of forces that would attract objects around the black hole and that this would interfere with the passage of time.

This conception is more evident in the gesture #2HOLE2 (Figure 7.34) performed by the student, where she moved her right hand, closing it, as if pulling something, to indicate the black hole "sucking" objects around it. The interpretation of gravity as the geometry of spacetime proposed by GR conflicts with many students' daily experiences, so it is common for them to associate "gravity as a force" (Steier; Kersting, 2019).

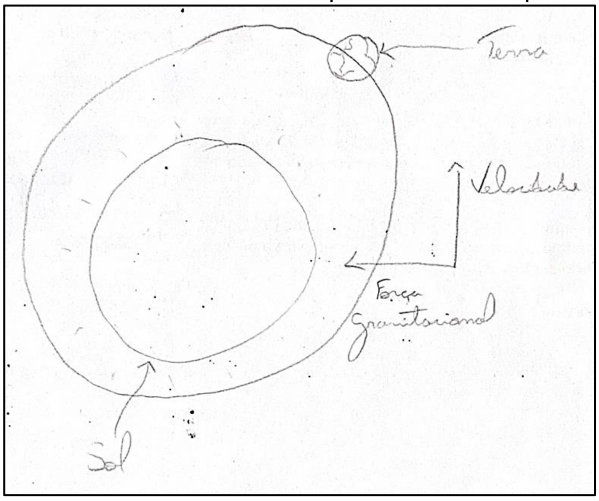
7.3 LIMITED UNDERSTANDING AND INCONSISTENT MENTAL REPRESENTATIONS

Students with limited understanding were unable to correctly answer many questions on the tests, nor did they provide coherent explanations for their responses. Among these students is Mike, who will be presented here.

7.3.1 Curved Spacetime

In his answer to question 9 on the post-test, regarding the Earth's orbit, Mike did not demonstrate to establish any connection with the curvature of spacetime. In the drawing provided by the student, he mentioned “gravitational force” and the “velocity” of the Earth's movement (Figure 7.35).

Figure 7.35: Mike's answer to question 9 of the post-test.



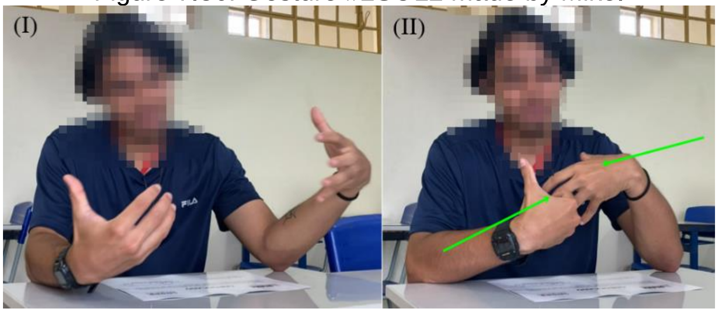
Source: Author (2024).

During the interview, Mike explained his response, demonstrating an association between the Earth's movement and the force exerted by the Sun on it:

> I put gravity; I think to represent the question of gravitational force. Because the Sun is pulling it [Earth] [#2SOL2 31:37], right? I put velocity too, because I think the Sun doesn't get too close [#2SOL2 31:51], it's not… the Sun tries to get closer [#2SOL2 31:54] to things near it... It [Earth] maintains that trajectory, it gets close [#2ORB3 32:09], then it passes by far.

As Mike mentioned in his explanation, he associated the Earth's movement with the fact that the Sun is “pulling” it. The student even performed the gesture #2SOL2 (Figure 7.36), where he moved his two hands forward, indicating that force of attraction. Then, Mike mentioned the Earth's trajectory as a consequence of that force and performed the gesture #2ORB3 (Figure 7.37), where he moved his right hand in a counterclockwise orbit, indicating the Earth’s orbit. As mentioned earlier, it was anticipated that some students would face difficulty in changing their conception of “gravity as a force”, since the interpretation of gravity as spacetime geometry “contradicts” many of their daily experiences (Steier; Kersting, 2019).

Figure 7.36: Gesture #2SOL2 made by Mike.



Source: Author (2024).

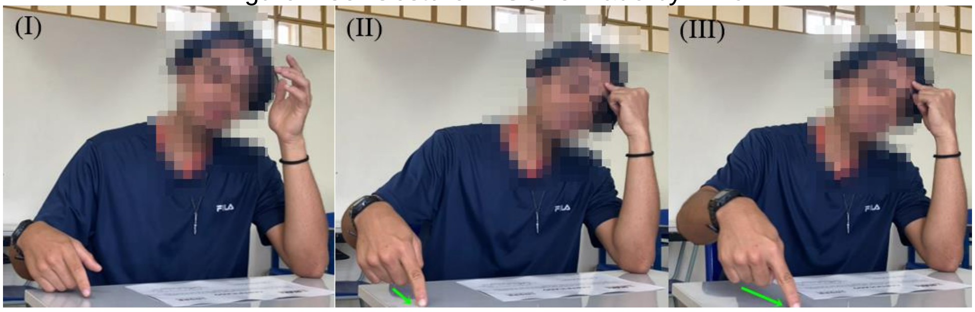
Figure 7.37: Gesture #2ORB3 made by Mike.

Source: Author (2024).

Both Mike’s gestures and description demonstrated that he possesses a pictorial mental representation for the force exerted by the Sun and the Earth’s movement in orbit around it. The fact that Mike mentioned velocity also reinforces that he has focused on the trajectory performed by the Earth around the Sun. When asked what he remembered to answer the question, Mike mentioned videos of simulations he had already watched on Instagram.

> I’ve seen a lot of simulations about this [planets’ movements]. I’ve seen even a simulation [observing] from outside, showing the Sun going [#2SOL3 32:32] at some velocity, along with all [#2PLAN2 32:34] the planets that orbit it [...] It’s on Instagram, pages which I follow, like the “Quantum Humor” page, and Pedro Loos too.

Figure 7.38: Gesture #2SOL3 made by Mike.



Source: Author (2024).

Figure 7.39: Gesture #2PLAN2 made by Mike.

Source: Author (2024).

While describing the video, Mike performed the gesture #2SOL3 (Figure 7.38), moving his right index finger forward over the table, indicating the Sun's movement, and the gesture #2PLAN2 (Figure 7.39), where he positioned his left hand below and his right hand above, and moved both, rotating them counterclockwise, indicating the movement of the planets around the Sun.

These gestures reinforce that Mike has developed a pictorial mental representation for the movement of the Earth and other planets, but not for the cause of it, namely, the spacetime curvature. Possibly, the interaction with this hypercultural resource, the video on the Internet, has assisted Mike in developing *drivers* in the form of a mental representation of the Earth's movement.

7.3.2 Gravitational Time Dilation

Regarding gravitational time dilation, Mike presented a satisfactory understanding of the phenomenon. In his response to question 7 of the post-test, where one astronaut went down to Earth and the other waits in the space station, Mike marked "a little more than 8 h" for the astronaut in the space station, indicating some comprehension.

> I imagined the situation from the movie [Interstellar]. The one [scene] with a planet near the event horizon of a black hole. And then, analysing, I thought it was, like, the more massive the object, the slower time will pass. Then, since

> we're... Since there's one [astronaut] that's outside the Earth, I think he is totally outside the Earth's cycle and will end up aging more.

Although Mike mentioned the "Earth's cycle", he also associated the fact that time slows down near a massive object. It is interesting to note that, unlike what other students presented, Mike seems to have recognized Earth as a massive object capable of causing changes in the passage of time. He mentioned the scene from the movie *Interstellar*, where the characters visit a planet near a black hole. Possibly, the interaction with this cultural resource helped Mike to understand the phenomenon and transpose the situation from the movie to the test question.

Regarding question 8 of the post-test, a similar situation to the previous question, but involving a black hole, Mike also responded correctly that it would take "much more than 3 h" for the astronaut who waited in the spaceship. Again, the student mentioned the movie scene to explain his response.

> It was also the situation from the movie, from the part with the black hole, it's much more than 3 h for the one [astronaut] in the spaceship. Because for the one on the planet near the massive object, time is slower. Then for the one outside that massive object, it will pass much more time, more than 3 h. [...] The larger the object [#2MASS3 30:19], the slower time passes.

Figure 7.40: Gesture #2MASS3 made by Mike.

Source: Author (2024).

As stated previously, Mike related the passage of time to being near a massive object. The student still mentioned that "the larger the object, time passes more slowly". Therefore, it is possible to observe that, once again, the student was able to transpose the situation observed in a cultural resource to solve the test question. Although Mike explained his response by performing the static gesture #2MASS3 (Figure 7.40), where he positions his two open hands in front of him, this gesture represents the massive object, not the passage of time itself. Therefore, possibly Mike developed *drivers* in the form of a propositional mental representation for gravitational time dilation through his interaction with the cultural resource of the movie.

8 RESULTS COHORT 3

The ten participants in Cohort 3, similar to those in Cohort 1 and Cohort 2, were divided into three groups. Consequently, the results are presented according to these groups: satisfactory understanding and consistent mental representations (3); partial understanding and developing mental representations (2); and limited understanding and inconsistent mental representations (5).

The following chapter presents and discusses the cases of five students, half of the sample: two with satisfactory understanding, one with partial understanding, and two with limited understanding. Within each group, the conceptions and mental representations of the students are analysed for each key concept worked on.

As this experiment focused on learning about General Relativity Theory, the conceptions and representations regarding curved spacetime and gravitational time dilation were analysed. The foundation for understanding GR consists of the idea of a four-dimensional curved spacetime (Kersting, 2019b; Mcinerney; Sutton, 2024), which is quite abstract and non-intuitive. Therefore, a greater focus was given to the analysis of conceptions and representations regarding curved spacetime. This analysis primarily led to the categorisation of students into the three groups.

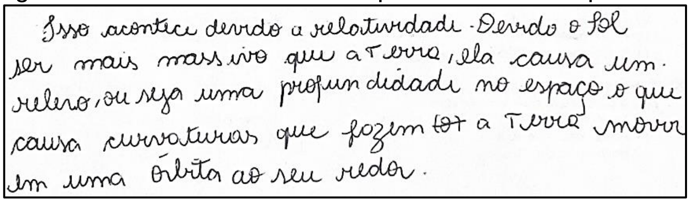
8.1 SATISFACTORY UNDERSTANDING AND CONSISTENT MENTAL REPRESENTATIONS

In general, the students in this group demonstrated a good understanding of curved spacetime, linking it to the explanation of various phenomena. Consequently, they were able to answer many test questions correctly, providing elaborate explanations during the interviews. The students Ana and Sam demonstrated a correct understanding of the rubber-sheet analogy, presenting evidence of robust mental representations of curved spacetime in the form of vivid images.

8.1.1 Curved Spacetime

Question 11 of the test addressed the reason why the Earth orbits the Sun: “Explain, as you were telling to a classmate, why the Earth moves in an orbit around the Sun. To do that, you can use text, drawing, graphics, diagrams etc”. In her post-test response, Ana mentioned that the curvature was caused by the mass of the Sun: “This happens due to the relativity [of the masses]. Because the Sun is more massive than Earth, it causes a shape, a depth in space that causes curvatures that make the Earth move in an orbit around the Sun”.

Figure 8.1: Ana’s answer to the question 11 of the post-test.



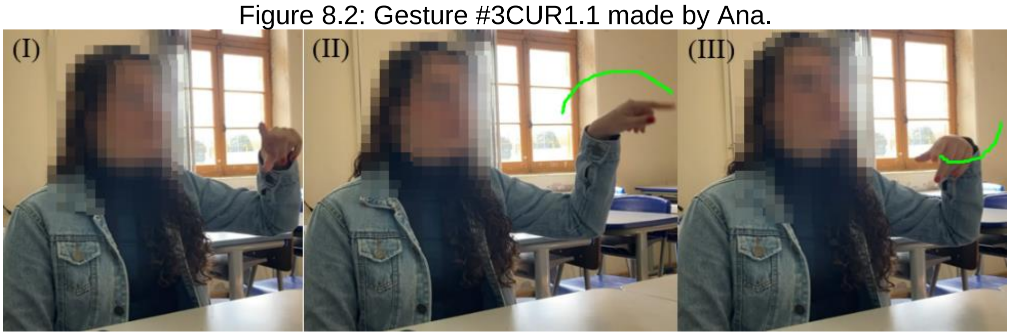
Isso acontece devido a relatividade. Devido o Sol ser mais massivo que a Terra, ela causa um relevo, ou seja uma profundidade no espaço o que causa curvaturas que fazem a Terra mover em uma órbita ao seu redor.

Source: Author (2024).

In her text, Ana related the movement of the Earth to the curvature caused by the mass of the Sun. She used the word “depth”, which expresses an idea similar to “deformation”, but brings the idea of “down”. Possibly Ana related this to the rubber-sheet analogy representing the effects of curvature caused by massive objects going down, a limitation of this model (Postiglione; De Angelis, 2021b). During the interview, Ana was able to explain her response and relate it to the activities in class:

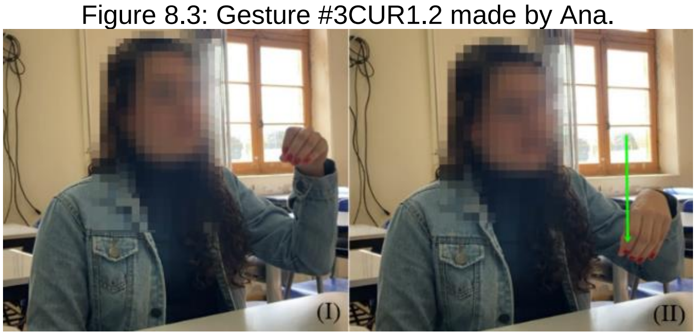
> [...] that experiment with the marbles came to mind to answer this one [question 11], because I hadn’t thought of it, so the reasoning was quite different from the other one [pre-test] when I didn’t know about relativity, for example, I didn’t know that it [Sun] causes a curvature [#3CUR1.1 25:36] due to mass. So I put that answer [points to the pre-test]. This [post-test] I had already taken into account this scenario. The experiment we did and due to massive objects, that it [mass] really causes [#3CUR1.2 25:47], so I imagined that this could [be the] cause [#3CUR1.3 25:49] to [why] Earth goes around [3CUR1.3 25:52] the Sun.

Figure 8.2: Gesture #3CUR1.1 made by Ana.



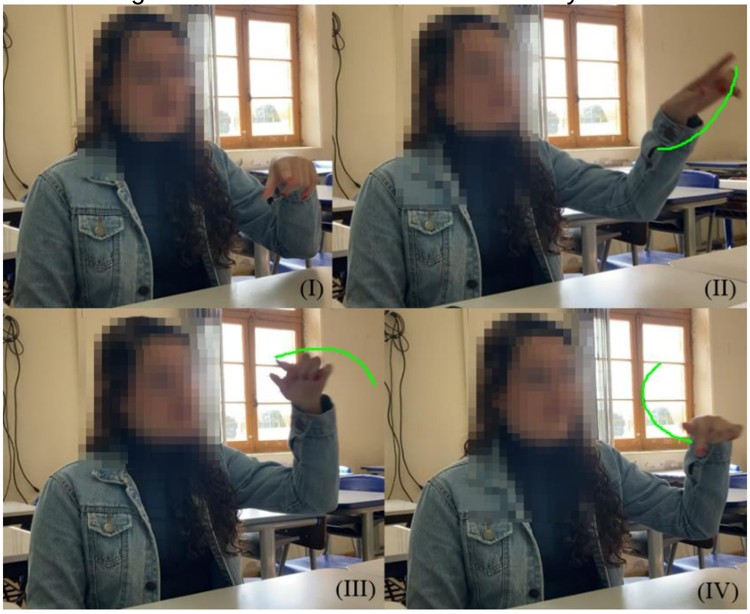
Source: Author (2024).

Figure 8.3: Gesture #3CUR1.2 made by Ana.



Source: Author (2024).

Figure 8.4: Gesture #3CUR1.3 made by Ana.



Source: Author (2024).

Through Ana’s explanation, it can be observed that she was able to transpose a physical example presented in class (the rubber-sheet model) to a new situation, the reason why the Earth orbits around the Sun. When Ana mentioned the curvature caused by the Sun, she performed gesture #3CUR1.1 (Figure 8.2), moving her left hand in an anticlockwise orbit. This dynamic gesture, aligned with her speech, are indicators that she has a mental simulation for the Earth’s movement that is related to that curvature.

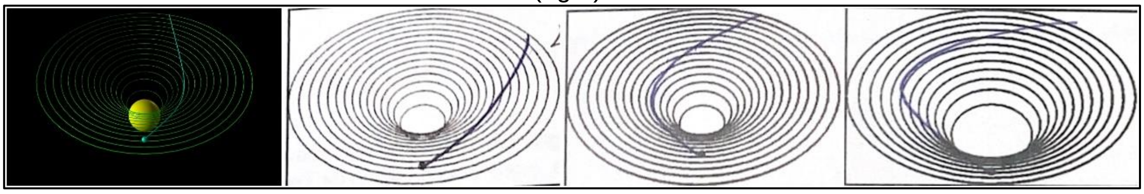
Ana mentioned the rubber-sheet model directly, performing gesture #3CUR1.2 (Figure 8.3), moving her left hand down to represent the deformation, or in her words, the “depth” caused by the balls in the fabric. Finally, she connected the Earth’s orbit to the rubber-sheet model, performing gesture #3CUR1.3 (Figure 8.4), also suggesting a connection between her mental representation, identified earlier by gesture #3CUR1.1 and her discourse, and the rubber-sheet model.

Following the interview, Ana also mentioned a hypercultural resource, the simulation “Embedding Diagrams” (Figure 8.5):

> [I remembered] this activity [looks at the simulation script] when we threw an object close [to a massive object] and we had to [#3EMB1.3 26:26] make a line, [#3EMB1.4 2x 26:28] so we had an idea of this curvature around it [massive object].

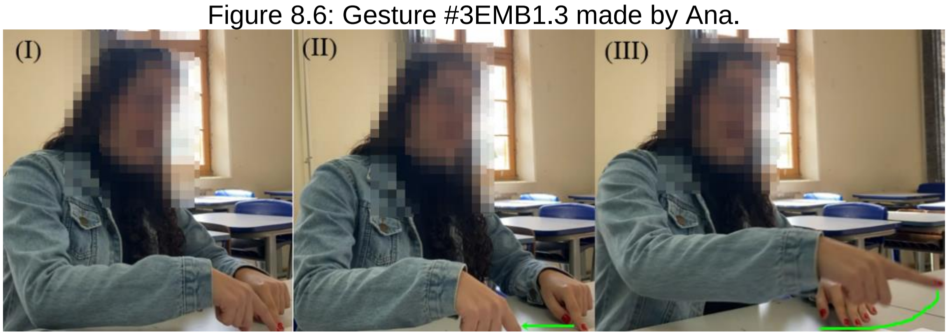
In this passage, Ana performed the gesture #3EMB1.3 (Figure 8.6), moving her right index finger drawing a curved line to represent the diagrams she drew during the activity with the “Embedding Diagrams” simulation. Therefore, Ana related this activity to the Earth's orbit, as she mentioned "the curvature" performing gesture #3EMB1.4 (Figure 8.7), in which she moves her right hand in a anticlockwise orbit, similar to gesture #3CUR1.1 performed when explaining the Earth's orbit around the Sun. These are indicators that Ana used the same mental representations as before to answer the post-test question.

Figure 8.5: Screen of the simulation "Embedding Diagram (left) and Ana's drawings during the activity (right).



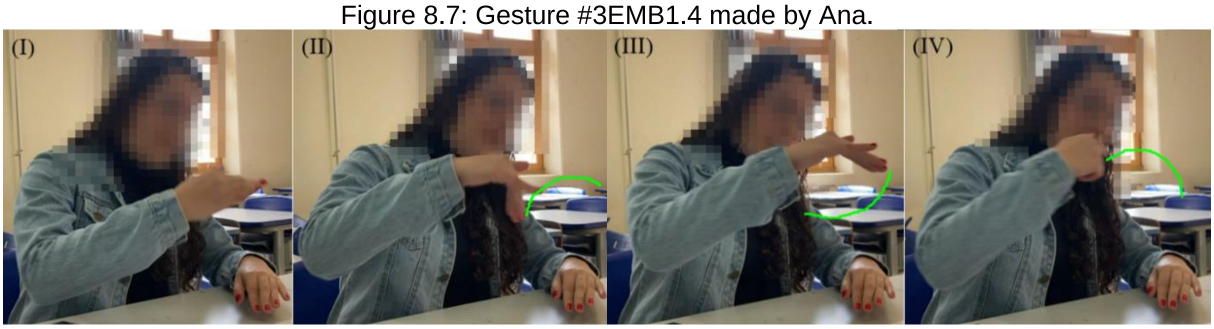
Source: Author (2024).

Figure 8.6: Gesture #3EMB1.3 made by Ana.



Source: Author (2024).

Figure 8.7: Gesture #3EMB1.4 made by Ana.

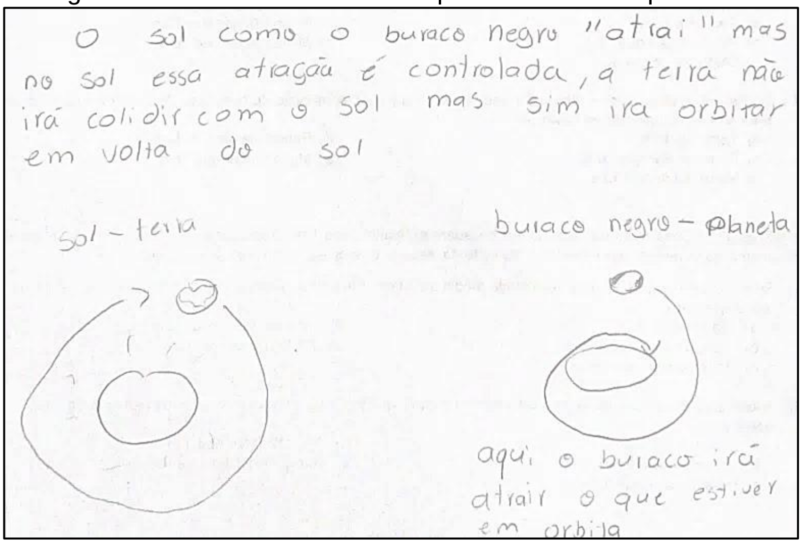


Source: Author (2024).

Analysing Ana's gestures and discourse, it can be observed that the student was able to imagine the curvature caused by a massive object, in this case the Sun. These are evidence that Ana was able to internalize *drivers* due to interaction with these external organized systems, through psychophysical and hypercultural mediation, which influenced her cognitive processes. These *drivers* were developed in the form of pictorial mental representations, mental simulations, which Ana used in her reasoning to answer the proposed question. The fact that Ana used different external resources to develop her mental representations highlights the importance of complementing the rubber-sheet model with other resources such as simulations (Kersting; Steier, 2018).

The student Sam presented similar ideas about curved space-time for the same question on the test, question 11. Analysing her post-test response, it is possible to note her difficulty in expressing her thoughts verbally. However, Sam used depictive gestures to communicate her ideas during the interview and drawings comparing the effects caused by a black hole to those caused by the Sun. Her textual answer: “The Sun, like a black hole, ‘attracts’, but in the Sun, this attraction is controlled, the Earth won’t collide with the Sun, but will orbit around it”.

Figure 8.8: Sam’s answer to the question 11 of the post-test.

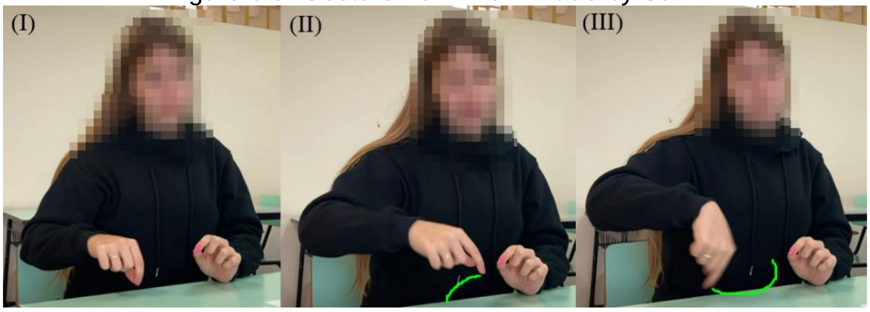


Source: Author (2024).

In the text and drawing response, it is not possible to note clear indications of ideas about curved spacetime. On the contrary, the response seems to be related to the idea of a force that “pulls” or attracts objects. However, during the interview, Sam explained, mainly using depictive gestures, that she tried to represent what she observed during the activities with the physical rubber-sheet model:

> [...] here [post-test] I remembered that experiment with the marbles [#3MAL5.1 2x 22:38]. You told us that as much heavier [#3MAL5.2 22:42], as bigger the mass, more it will be [#3MAL5.3 22:44] [...] that's what I tried to explain, that the Sun will have that thing I don't remember the name, so it [Earth] won't collide with the Sun. And, if it was a black hole, it would collide, because the black hole attracts [#3BUR 23:08]. [...] I remembered of the experience, we were throwing the marbles [#3MAL5.4 23:20] and they were doing like this. I tried to represent it in the drawing [points to the drawing] making [#3MAL5.4 23:23], like the marbles did.

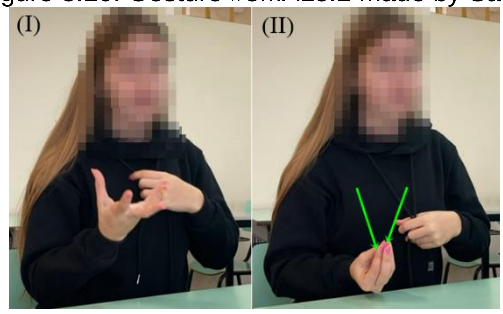
Figure 8.9: Gesture #3MAL5.1 made by Sam.



Source: Author (2024).

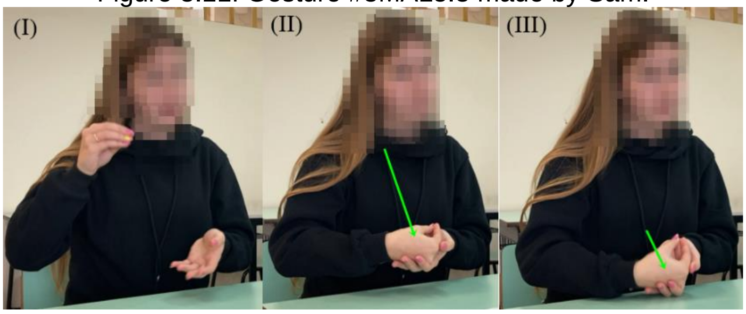
Analysing Sam's discourse and gestures together, it is possible to see that she was trying to represent the curvature observed in the rubber-sheet model. Several times she struggled to find the right word to express her ideas, using gestures to communicate them. When mentioning the marbles, Sam performed gesture #3MAL5.1 (Figure 8.9), moving her right hand in a clockwise orbit to describe the movement of the marbles, relating what she observed in the rubber-sheet model to the Earth's orbit.

Figure 8.10: Gesture #3MAL5.2 made by Sam.



Source: Author (2024).

Figure 8.11: Gesture #3MAL5.3 made by Sam.



Source: Author (2024).

Analysing Sam's discourse and gestures together, it is possible to see that she was trying to represent the curvature observed in the rubber-sheet model. Several times she struggled to find the right word to express her ideas, using gestures to communicate them. When mentioning the marbles, Sam performed gesture #3MAL5.1

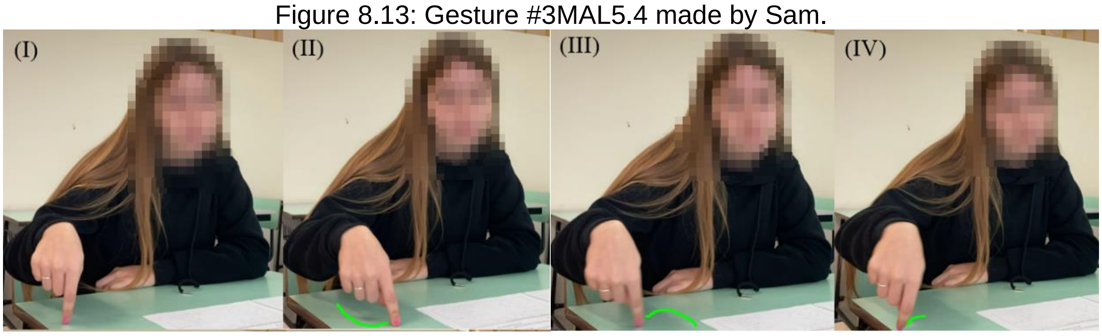
(Figure 8.9), moving her right hand in a clockwise orbit to describe the movement of the marbles, relating what she observed in the rubber-sheet model to the Earth's orbit.

To describe deformation caused by mass, Sam performed the gestures #3MAL5.2 (Figure 8.10), moving her right hand down and closing it, and #3MAL5.3 (Figure 8.11), moving her right hand down to meet her left hand, similar to Ana's gesture #3CUR1.2. These gestures reinforce the limitation of the rubber-sheet model, which depends on Earth's gravity to work, inducing students to have an idea of deformation downwards, as reported in other studies (Postiglione; De Angelis, 2021b).

Figure 8.12: Gesture #3BUR made by Sam.

Source: Author (2024).

Figure 8.13: Gesture #3MAL5.4 made by Sam.



Source: Author (2024).

Comparing it to a black hole, Sam performed gesture #3BUR (Figure 8.12), moving her right hand over the table towards her chest, indicating the strong attraction that would not allow an object to describe an orbit around it. Finally, Sam performed gesture #3MAL5.4 (Figure 8.13), making an anticlockwise orbit with her right index finger to describe the movement of the marbles she tried to represent in her drawing. This gesture, resembling the gesture #3MAL5.1 where she described the movement of the marbles observed during the lesson, reinforces the connection between her response and the in-class activity using the rubber-sheet model – psychophysical mediation.

As indicated by Sam's depictive gestures and discourse, she was able to transpose what she saw previously in the rubber-sheet model during the classes to a new situation. Like Ana, it was possible to identify indicators that Sam constructed *drivers* in the form of pictorial mental representations by interacting with a psychophysical resource – the rubber-sheet physical model – and accessing these *drivers* to answer the question 11 in the post-test.

These students also presented conceptions of curved spacetime in their AI-generated images regarding the word "relativity". The curvature caused by massive objects was mentioned by Ana in both her prompt and during the interview. Ana's prompt: "When a massive object with a bigger weight is in space, it causes a curvature, due to relativity, and as bigger the mass of this object or phenomenon in space, the bigger also the curvature will be, as if it was marbles in a sheet in an arc".

Figure 8.14: Ana's final AI-generated image.

Source: Author (2024).

Similarly, regarding the question 11 of the test, when discussing this activity in the interview, Ana mentioned the idea of curvature as a "depth". Although the generated image (Figure 8.14) was not so precise as Ana had hoped, what she was trying to represent, as evidenced by her prompt and discourse in the interview, demonstrates her conceptions of curved spacetime:

> Here, I think it's different because I added the depth [#3PRO 29:09] [...] and it's like this is relativity and it [massive object], is causing that depth in space. [...] I wanted this deformation. It wasn't exactly as I imagined, because when I imagined it, it was like a [#3RED1.1 29:29] grid, in my mind. It's something [#3PRO 29:32] causing this curvature [#3RED1.2 29:33] [...] This one, this grid [GIF slide 70], that's what I imagined.

Explaining her AI-image, Ana performed the gesture #3PRO (Figure 8.15), moving her right hand downwards towards her left hand, which resembles the gesture #3CUR1.2 she performed when discussing the rubber-sheet model. This similarity between these gestures suggests that Ana was trying to represent in the generated image the same mental representation she used to answer the question 11 of the test.

Figure 8.15: Gesture #3PRO made by Ana.

Source: Author (2024).

Figure 8.16: Gesture #3RED1.1 made by Ana.

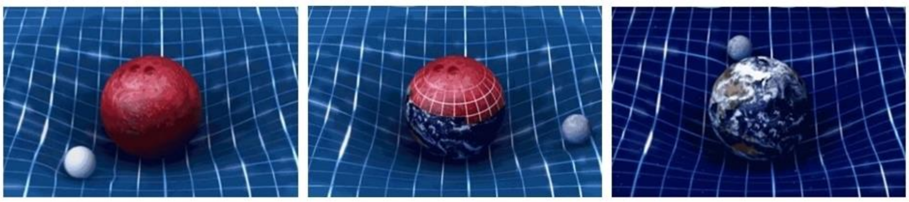
Source: Author (2024).

Figure 8.17: Gesture #3RED1.1 made by Ana.

Source: Author (2024).

According to her description, Ana tried to represent the deformation in a grid, similar to the representation depicted in the GIF used in the slides (Figure 8.18), as confirmed by her. The student performed the gesture #3RED1.1 (Figure 8.16), where she places her two hands together and open in front of her chest and moves them horizontally one to each side, and the gesture #3RED1.2 (Figure 8.17), where she places her two hands together and open in front of her chest and moves them in a curve upwards. Both gestures were used to represent a surface curved by a mass.

Figure 8.18: GIF used in slide 70 from presentation at three different moments.



Source: Author (2024).

These gestures, aligned with the student's descriptions, are indicators that Ana probably developed *drivers* (mental pictorial representations) through interaction with this hypercultural resource (GIF). This mental representation was used by the student in her image generation process with AI.

Sam presented similar ideas in her image generation process. However, as before, the student had difficulties in describing her thoughts verbally. Sam's prompt was ambiguous and contradictory: "I think of a plane deformed sphere comparing to the Earth". Both the prompt and the resulting image (Figure 8.19) do not provide clear information about the student's conceptions.

Figure 8.19: Sam's final AI-generated image.

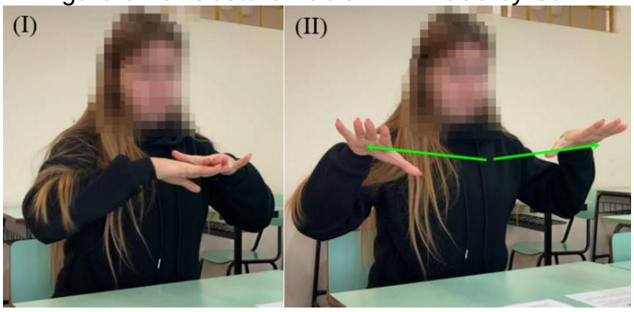
Source: Author (2024).

However, during the interview, Sam performed many gestures to externalize and communicate her reasoning and what she was trying to represent, showing her conceptions of curved spacetime:

> [...] [Relativity] would be a trampoline [#3CUR2.1 26:03], but the AI didn't understand what I wanted... And the trampoline is kind of bent [#3CUR2.2 26:08] because the black hole deforms it as shown on this trampoline [#3CUR2.1 26:12], as I imagined. But then I couldn't describe this [deformation], so I think I wrote that it's a plane deformed sphere compared to the Earth. [...] It [AI] didn't understand me, and I also couldn't express what I was imagining. Because, as I told you, I was imagining like bent [#3CUR2.3 27:04], and I wasn't able to express.

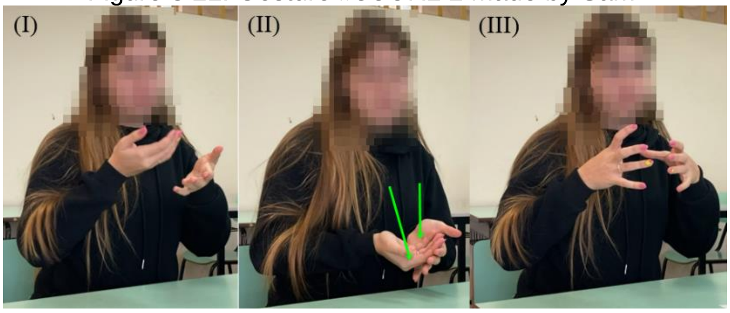
Sam's discourse indicates that, when she thought about the word "relativity", she imagined the rubber-sheet analogy, but using an trampoline. However, as Sam reported, the resulting image did not represent her mental image, as she was unable to express herself adequately.

Figure 8.20: Gesture #3CUR2.1 made by Sam.



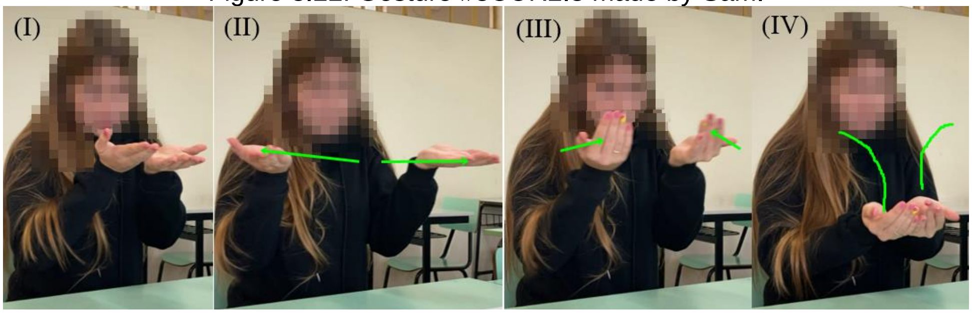
Source: Author (2024).

Figure 8.21: Gesture #3CUR2.2 made by Sam.



Source: Author (2024).

Figure 8.22: Gesture #3CUR2.3 made by Sam.



Source: Author (2024).

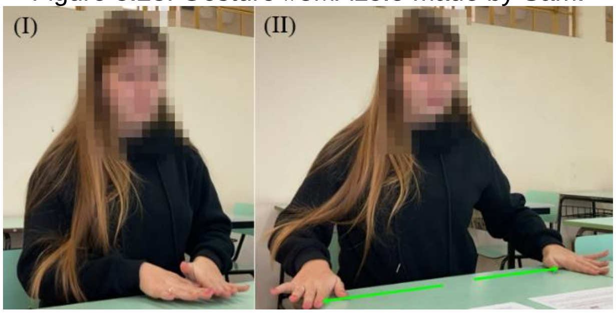
It is possible to note the connection with massive objects when Sam performed the gestures #3CUR2.1 (Figure 8.20), placing her two hands open together in front of her chest and moving them horizontally one to each side, and #3CUR2.2 (Figure 8.21), moving her hands together downwards and positioning them in a "C" shape, representing a sphere. The gesture #3CUR2.1, in turn, resembles the gesture #3MAL5.2 performed earlier by Sam, indicating that she probably used the same mental representation constructed through interaction with the rubber-sheet model.

The idea of curvature, or, as Sam said, “bent”, is observed in her gesture #3CUR2.3, where she moves her two hands horizontally, then moves them in a curve downwards and towards the centre, representing the curved surface of the trampoline she imagined using gestures to communicate the expected result. Possibly, the black object next to the Earth in the AI-generated image is the AI’s attempt to represent the "plane deformed sphere" (a possible description could be "a plane deformed by a sphere" representing the "curved trampoline").

Following the interview, Sam mentioned the rubber-sheet model again:

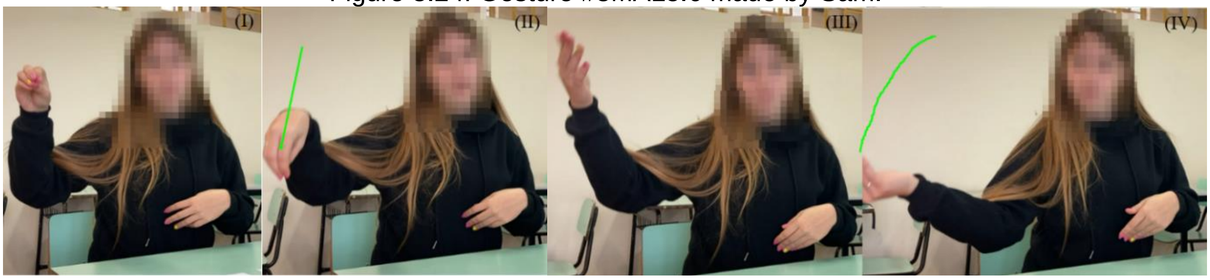
> [I remembered] about that experience of the marbles on the fabric, that you told us the fabric was [#3MAL5.5 27:20] the plane, and then when you put some mass, a weight, it started being deformed [#3MAL5.6 27:25]. And also, the Internet images about relativity, when I searched for it.

Figure 8.23: Gesture #3MAL5.5 made by Sam.



Source: Author (2024).

Figure 8.24: Gesture #3MAL5.6 made by Sam.



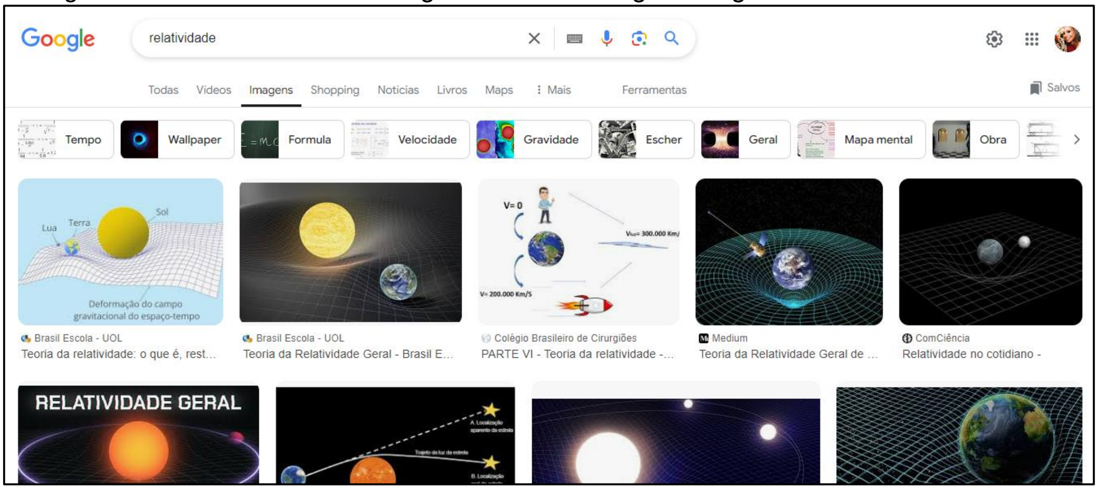
Source: Author (2024).

Sam probably developed her mental representation of the "trampoline" described by her from interaction with the rubber-sheet model. She performed the gesture #3MAL5.5 (Figure 8.23), where she placed her two hands open together on the table and moved them horizontally one to each side, resembling part of the gesture #3CUR2.2 used to represent the trampoline earlier, and also directly mentioned the activity with this psychophysical resource from class.

The idea of curvature created by the analogy is also observed in the gesture #3MAL5.6 (Figure 8.24), where she moved her right hand downwards and backwards in a curve when mentioning the deformation. As both gestures, #3MAL5.5 and #3MAL5.6, suggest that she associated this deformation with downwards, as mentioned earlier, this reflects the limitation of the physical rubber-sheet model used by her to construct her mental representation.

Sam also mentioned images found on the Internet when researching "relativity", highlighting the hypercultural mediation level. The usual resulting images from this search represent the rubber-sheet analogy (Figure 8.25), similar to the GIF used in the slide 70 mentioned by Ana. Considering the similarities of ideas and gestures (#3RED1.1 and #3CUR2.3) presented by the two students, it is possible to infer that Sam probably used images or GIFs, found on Internet searches or even in the slides (hypercultural mediation), to construct her mental representation for curved spacetime.

Figure 8.25: Results from a Google search for images using “relatividade” at 04/10/24.

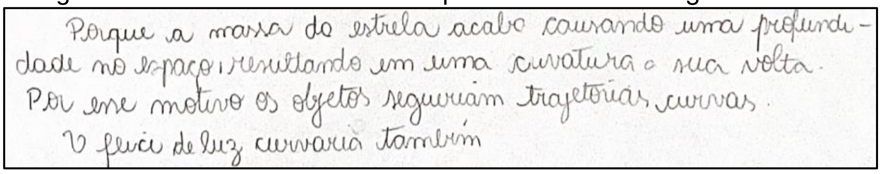


Source: Author (2024).

Another evidence of Ana and Sam’s understanding of curved spacetime can be observed in their answers to an exercise solved in class that deals with the phenomenon of light bending. Question 3 of Exercise List 3 asked: “Consider the star Sirius, with a mass more than twice the Sun’s mass. Why would objects near it describe curved trajectories? Would a light beam passing nearby be curved? Explain (you can use text, drawings, diagrams, etc.)”.

Ana answered by text that “The star’s mass causes a depth on space, resulting on a curvature around it. Because of it, the objects would follow curved paths. The light beam would be curved too” (Figure 8.26). The fact that the student used words like “depth” is an indication that, when solving a different problem (about light bent), Ana accessed the same mental representation used in previous situations, constructed through interaction with psychophysical and hypercultural mediations.

Figure 8.26: Ana’s answer for the question solved during the classes.

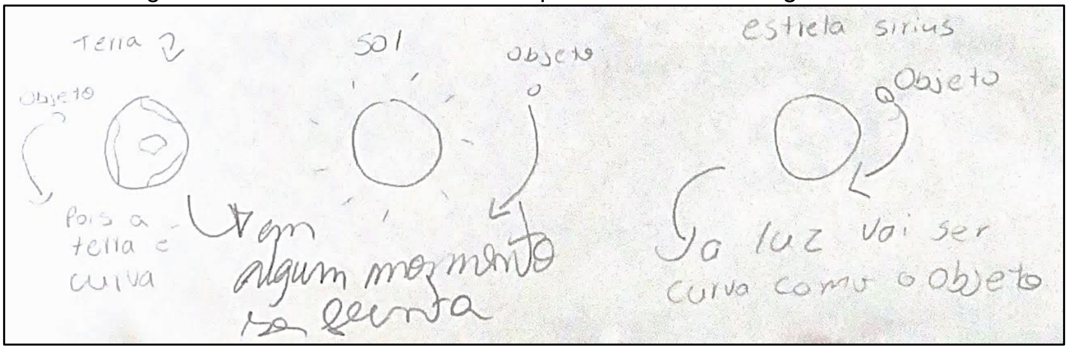


Source: Author (2024).

Similar ideas can be observed in Sam's answer, where she used short explanations and drawings – similar to those used for her answer to question 11 of the post-test – and compared the effects of the Earth and the Sun to the massive star Sirius (Figure 8.27). Again, even using drawings, Sam showed difficulties in expressing her ideas.

However, observing the similarities between the responses to this question and the test, it is possible to infer that she had a similar reasoning to Ana. Sam probably used her pictorial mental representation of curved spacetime; the *drivers* she acquired through interaction with the rubber-sheet model and images or GIFs on the Internet, as identified earlier.

Figure 8.27: Sam's answer for the question solved during the classes.



Source: Author (2024).

These results indicate that both students may have used different external resources through different levels of mediation, highlighting the psychophysical and hypercultural levels, to construct their pictorial mental representations of curved spacetime. Ana and Sam accessed these simulations or mental images to help them understand different situations proposed, responding correctly to many test questions and providing coherent explanations during the interview.

8.1.2 Gravitational Time Dilation

Regarding gravitational time dilation, Ana demonstrated a good understanding. Question 7 of the test asked: "Two astronauts are in the International Space Station. Consider that one of them goes down to Earth, do an activity and finished it in 8 h, according to his watch. What was the time interval for the astronaut who stayed in the space station?". Ana selected the answer "much more than 8 h". Although the correct answer was "little more than 8 h", the student demonstrated a good understanding of the phenomenon when explaining her choice:

> […] I think that because he is on Earth, it will be more... Like we studied that part about mass [#3MAS2 16:27], about the more massive objects [...] So, I think he would, the person there [space station], would pass more time than the person who is there on Earth.

In this excerpt, it is possible to observe that Ana established a relationship between the mass of an object, such as Earth, and the passage of time. This association is reinforced by gesture #3MAS2 (Figure 8.28), where Ana moves her two hands downwards in front of her, representing a massive object.

Figure 8.28: Gesture #3MAS2 made by Ana.

Source: Author (2024).

Regarding question 8, Ana's conceptions are more clearly observed. The question asked: "Consider now that the astronauts are on a spaceship close to a supermassive black hole. One of them goes in a mission and gets closer to the black hole, finishing the mission in 3 h, according to his watch. What was the time interval for the astronaut who stayed on the spaceship?".

Ana selected an incorrect answer, "much less than 3 h". However, during the interview, she corrected her answer, recognizing an error interpretation. Ana answered the question thinking that it referred to the time interval for the astronaut who approached the black hole (which would be the correct answer in this situation). But the question asked about the time interval for the astronaut who waited on the spaceship. Upon re-examining the question, Ana stated that it would take much more time for the astronaut who waited on the spaceship:

> Oh, so, in this question, I wouldn't choose the same answer today. I would put that for him [astronaut who stayed on the spaceship], it would take longer [...] I think much longer [...] I remembered when the teacher talked about black holes being very massive compared to Earth, the Sun, and sometimes we don't think about this. So, in the moment I was answering, here [post-test] I misinterpreted, but today I think about this.

Once again, Ana established a relationship between the time passage and massive objects. Although she had previously performed a gesture, #3MAS2, representing a massive object, the student did not perform other depictive gestures when explaining her answer. Ana also she did not state having any type of mental image for the situation, recalling the teacher’s explanations – therefore, social mediation level.

Ana was then asked about a new situation with an object of different mass, a star more massive than Earth and less massive than the black hole. Ana correctly answered that the difference in time would not be as pronounced as in the question: “It would be, I think, a bit more. If in the black hole I would answer much difference, this difference would be a bit less than in the black hole”.

Ana’s affirmation again demonstrates that she established a relationship between mass and the passage of time. Her explanation suggests that she correctly associated that the more massive the object, the more it would affect time. Once again, Ana did not perform depictive gestures or report any mental imagery processes. This absence of mental images, but with a coherent answer and explanation, suggests that she has developed a propositional mental representation for gravitational time dilation.

On the other hand, the student Sam did not demonstrate so clear understanding about the gravitational time dilation. However, as Sam showed a good understanding of the basis of curved spacetime, she was considered to have a satisfactory understanding. Dealing with question 7, Sam presented the misconception that “"in space, time passes more slowly”, selecting the answer “much less than 8 h” for the astronaut on the space station:

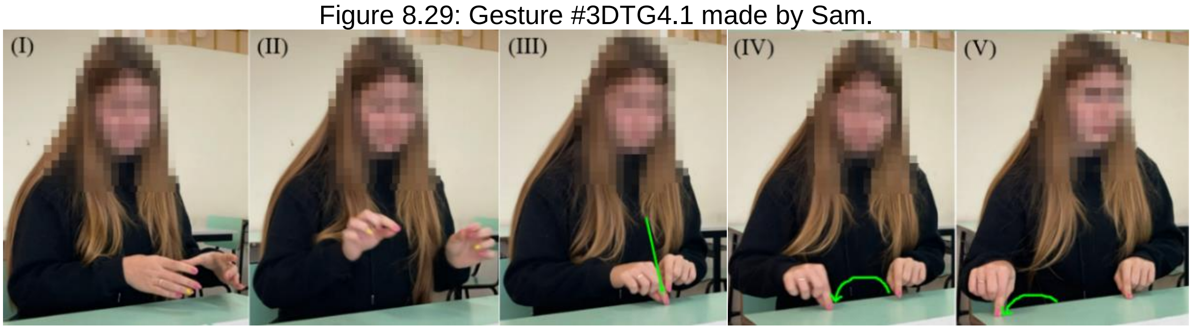
> [...] In space, time passes much more slowly. What confused me was the space station, because I don’t know what it is. But I tried to think because... As in the brothers’ situation, in the brothers’ question, time passed much more slowly for the brother who stayed in space. So, I used this idea to answer this question.

Sam related the question to an exercise done during the classes dealing with Special Relativity, not distinguishing between time dilation in SR and gravitational time dilation, a common error among students (Gousopoulos; Kapotis; Kalkanis, 2016). Possibly, she did not recognize Earth as a massive object capable of influencing the time passage, even minimally. Other studies have already reported that, even though students understood Einsteinian ideas, they have difficulty to connect these ideas to everyday life (Kersting; Henriksen; Bøe; Angell, 2018).

For question 8, the student also presented some misconceptions, but she demonstrated having an initial process of understanding of the phenomenon. Sam correctly associated that time would be slowed down near the black hole, but confused the proportion of the time difference:

> Here [post-test] I remembered that [simulation, points to the laptop], that you showed us the black hole [#3DTG4.1 15:50] and then the dots [...] Here [#3DTG4.2 3x 15:57] and here there wasn't much difference between one point and the other, because it's close to the black hole [#3DTG4.3 16:00], which is massive. Now, here [#3DTG4.4 16:02] passes a different time, which is a longer time than here [#3DTG4.5 16:05]. So, one [astronaut] was at a little distance from the other. It wasn't much distance [between the astronauts], so I answered that it would be a "little more than 3 h".

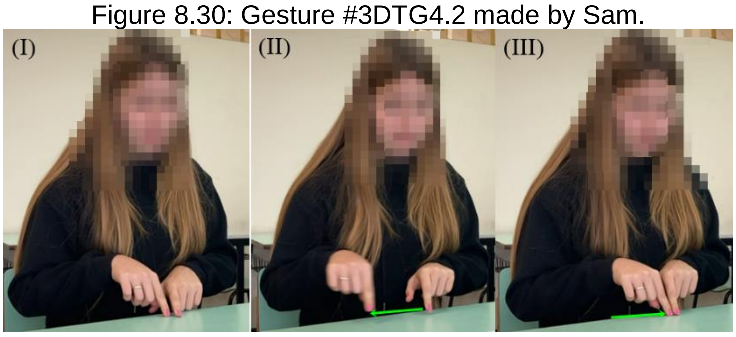
Figure 8.29: Gesture #3DTG4.1 made by Sam.



Source: Author (2024).

Sam mentioned remembering the simulation "Gravitational Time Dilation", performing gesture #3DTG4.1 (Figure 8.29) where she initially indicates the black hole simulation, representing a sphere with her two hands, and then indicates the three points at different distances from it. However, Sam demonstrated having incorrectly interpreted some information presented in the simulation.

Figure 8.30: Gesture #3DTG4.2 made by Sam.

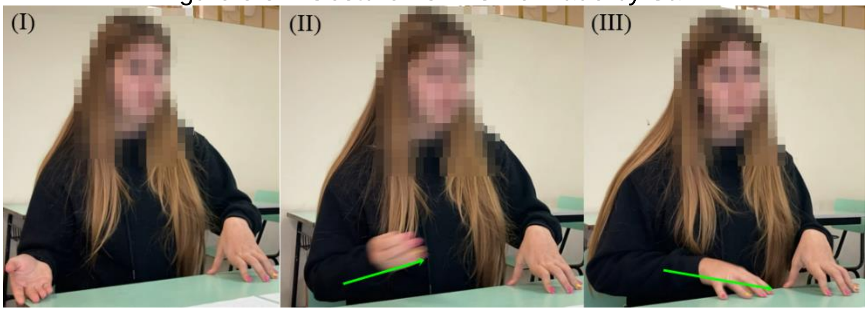


Source: Author (2024).

Figure 8.31: Gestures #3DTG4.3 (left) and #3DTG4.4 (right) made by Sam.

Source: Author (2024).

Figure 8.32: Gesture #3DTG4.5 made by Sam.

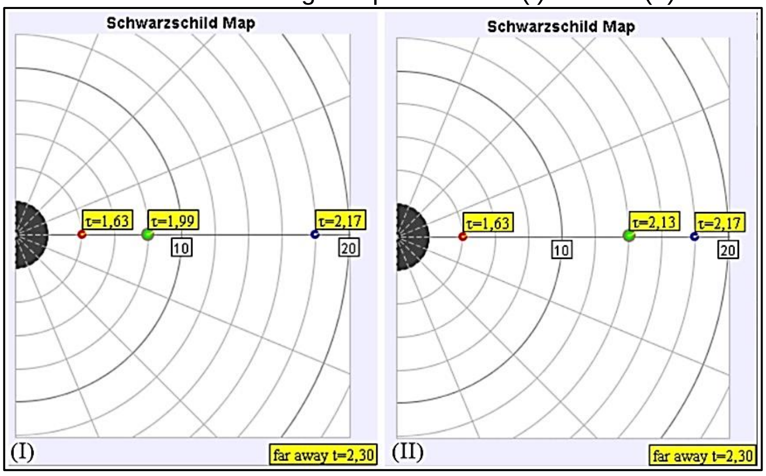


Source: Author (2024).

In the simulation, it is possible to observe the passage of time at three points at different distances from a black hole and compare it with the time outside the influence of this massive body (Figure 8.33). As it was possible to move these points, it was discussed in class that two points, maintaining their distance from each other, would have time differences becoming greater as they approached the massive object.

Sam, incorrectly, interpreted that the differences between the two points would be smaller closer to the black hole. Additionally, even though the question stated that one of the astronauts was approaching the black hole, the student considered the distance between the two astronauts small. Therefore, she considered that it would take only “a little more” time for the astronaut who waited on the spaceship.

Figure 8.33: Simulation screen showing two points close (I) and far (II) from the black hole.



Source: Author (2024).

Despite these erroneous ideas, Sam correctly understood that time would be slowed down near the black hole. When explaining her answer, Sam performed the gesture #3DTG4.3 (Figure 8.31, left) indicating the black hole with her left hand. Then, she performed gesture #3DTG4.4 (Figure 8.31, right) indicating a point at a certain distance from the black hole with her right hand, stating that the time in that point would be longer than the time at a point closer to the black hole, performing gesture #3DTG4.5 (Figure 8.32) bringing her right hand closer to her left hand – the black hole.

Observing the description and gestures performed by the student, it is likely that Sam possesses a pictorial mental representation still under construction for gravitational time dilation. This mental representation results from the activity performed with the “Gravitational Time Dilation” simulation. As this simulation was presented by the teacher on the projector, through discussions with the students, it is possible to affirm that Sam is using both the hypercultural and social mediation levels. However, since she incorrectly interpreted some information, she has not yet consolidated the *drivers* (mental representation, neither pictorial nor propositional) originating from this interaction.

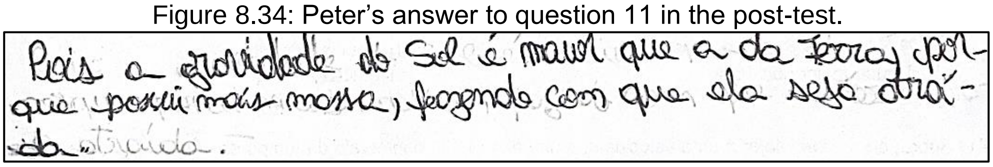
8.2 PARTIAL UNDERSTANDING AND DEVELOPING MENTAL REPRESENTATIONS

The students in this group demonstrated an initial, albeit limited, understanding of curved spacetime. Consequently, these students presented mental representations still under construction regarding the phenomena studied. Peter is a prime example of students from this group.

8.2.1 Curved Spacetime

Peter presented a partial understanding of curved spacetime. Considering question 11 of the test, dealing with the reason why the Earth moves in an orbit around the Sun, Peter’s answer appears more related to the concept of gravity as a force, rather than the geometry of spacetime: “Because the Sun’s gravity is greater than the Earth’s, due to its greater mass, making the Earth to be attracted”.

Figure 8.34: Peter’s answer to question 11 in the post-test.



Source: Author (2024).

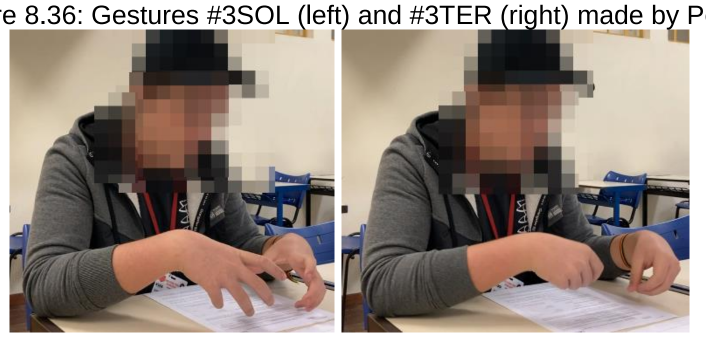
During the interview, Peter did not provide many additional explanations for his answer, nor did he demonstrate consolidated mental representations of the phenomenon, simply imagining the Sun and Earth with different sizes:

> [...] The Sun is much more massive than the Earth, it has much more mass, so the Sun's gravity is greater than the Earth's. Therefore, the Sun [#3ORB4 24:25] will make the Earth rotate around it. [...] I imagine, the Sun [#3SOL 24:42] much bigger, the Earth much smaller [#3TER 24:47], and therefore the Earth orbiting around the Sun.

Figure 8.35: Gesture #3ORB4 made by Peter.

Source: Author (2024).

Figure 8.36: Gestures #3SOL (left) and #3TER (right) made by Peter.



Source: Author (2024).

Although Peter demonstrated having an mental image for the Earth's orbit, as evidenced by gesture #3ORB4 (Figure 8.35), where he moves his right hand in a circular motion in a clockwise direction, he did not demonstrate having a mental representation related to the explanation of this phenomenon. The visible connection established by Peter was related to the Sun's mass, which is greater than the Earth's, as can be observed by gestures #3SOL and #3TER (Figure 8.36) representing the two bodies with different sizes.

Although Peter's ideas are not directly related to the interpretation of General Relativity's phenomenon, he correctly inferred the connection between the Sun's mass and the Earth's orbit. However, he presented a Newtonian idea, that this mass of the Sun would cause an attraction, and not the deformation of spacetime. Since the interpretation of gravity as spacetime geometry, as proposed by General Relativity, "contradicts" many daily experiences of students, it is common to consider "gravity as a force" (Steier; Kersting, 2019).

Considering Peter's AI-generated image (Figure 8.37), it was not possible to identify conceptions of curved spacetime at all. The student used the prompt: "A universe full with planets, stars, and black holes". During the interview, he stated:

> I took the word [relativity] and started thinking it in the sense of the universe itself. […] Luke told me to […] use the word 'relativity' not with its meaning itself […] this triggered me to make my image.

Figure 8.37: Peter's final AI-generated image.

Source: Author (2024).

Although he mentioned black holes, both in his explanation and in his prompt, Peter did not describe ideas of curved spacetime. However, in this excerpt it is possible to observe the influence of social mediation in Peter's image generation process. The discussion he had previously with a classmate helped him in the information process and influenced his ideas to generate the image.

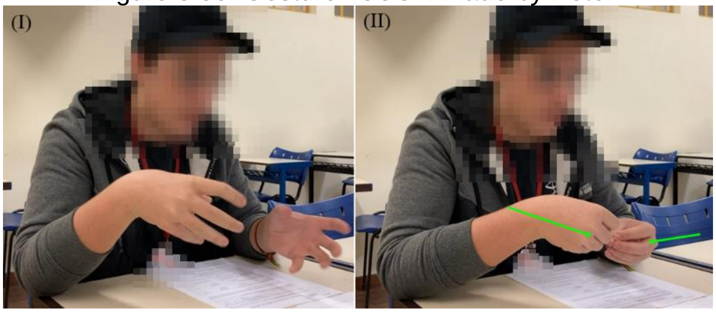
Although Peter did not present solid evidence of any understanding of curved spacetime when explaining his answer to question 11 of the test or the image generated with artificial intelligence, there were other moments during the interview where he demonstrated having an initial idea of curved spacetime.

Questions 9 and 10 of the tests dealt with the distortion of spacetime caused by a black hole: "9) Two points close to the Earth have between them a distance of 1 km. What would be the distance between these points if they were close to a black hole in an altitude of 500 km?". In question 10, the points would be 20 km from the black hole. While explaining his answers, Peter mentioned relating the proposed situation to the "fabric of the universe". Although he answered incorrectly that the distance between the two points would decrease near the black hole, he recognized the curvature caused by this massive body.

Peter's confusion, that spacetime would contract around the black hole, is likely due to the space contraction caused by speed of the reference frame, addressed by Special Relativity. Confounding SR and GR is a common difficulty among students (Gousopoulos; Kapotis; Kalkanis, 2016). As Peter mentioned:

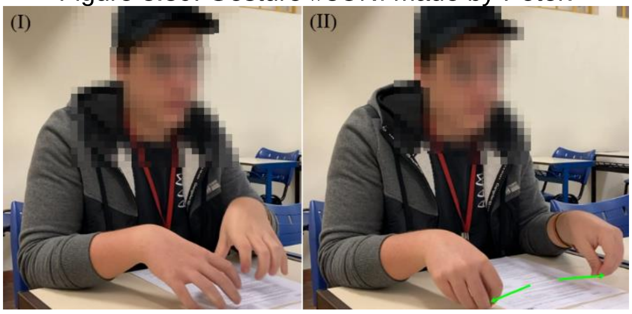
> I thought about space contraction. And then I thought that massive objects, they have a lot of influence on space. Then it [black hole] will make [#3CON 20:28] space contract [#3CON 20:31]. [...] I imagine the universe, it's a universe [#3UNI 22:47] full of stars, and then I imagine [#3UNI2 22:50] the fabric of the universe, the black hole in the middle, and it's like the fabric of the universe going in [#3MAL9.1 22:57]. Because of the influence of the black hole itself. [...] The closer to the black hole, the deeper [the space] [#3MAL9.1 23:08].

Figure 8.38: Gesture #3CON made by Peter.



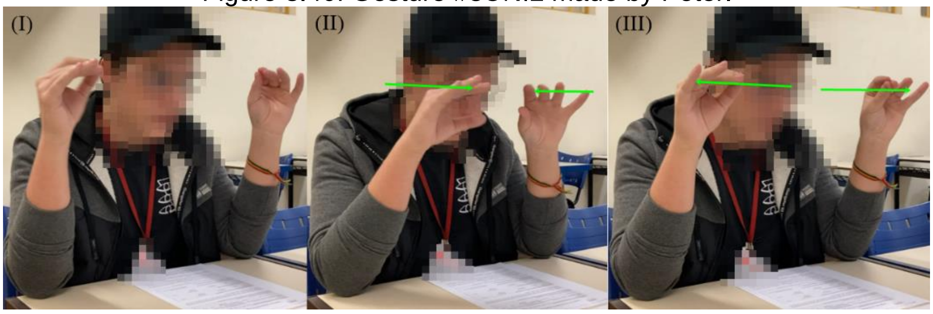
Source: Author (2024).

Figure 8.39: Gesture #3UNI made by Peter.



Source: Author (2024).

Figure 8.40: Gesture #3UNI2 made by Peter.

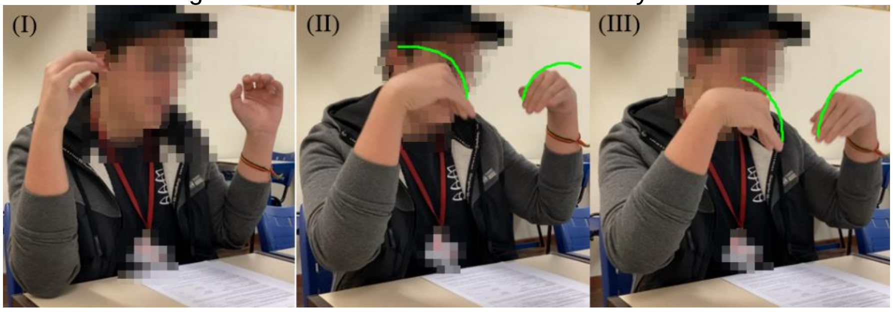


Source: Author (2024).

When speaking about space contraction, Peter performed the gesture #3CON (Figure 8.38), where he brings his two hands together, indicating this reduction in distance. However, when explaining what he imagined, the student mentioned the universe, performing the gesture #3UNI (Figure 8.39), and described the rubber-sheet analogy, referring to the "fabric of the universe" by performing the gesture #3UNI2

(Figure 8.40), where he moves his two hands horizontally, separating them. Then, Peter mentioned the black hole, performing gesture #3MAL9.1 (Figure 8.41), moving his two hands to the centre and downwards in a curved motion.

Figure 8.41: Gesture #3MAL9.1 made by Peter.

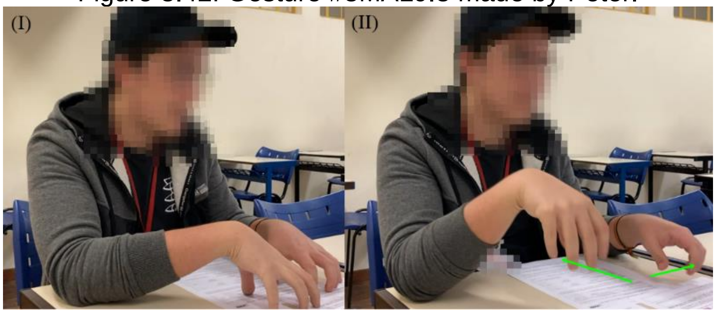


Source: Author (2024).

Although the student did not mention remembering any specific resource, the description provided by him and his depictive gestures are notably similar to those of Ana (#3RED1.2) and Sam (#3CUR2.3). This similarity suggests that Peter may have remembered the images and GIFs (hypercultural mediation), or even the rubber-sheet mode (psychophysical mediation) I used during the lessons, even if he did not associate his mental image with them consciously. Possibly these resources helped the student to process information about curved spacetime.

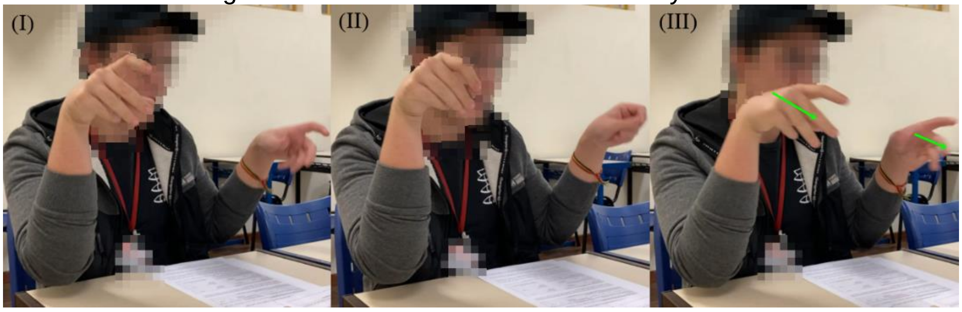
This possible connection with the psychophysical mediation can be observed in another moment during the interview, when Peter mentioned the rubber-sheet model, indicating that it was a significant resource for him: “When you brought that fabric [#3MAL9.3 28:47] with PVC pipes and then you threw the marbles [#3MAL9.4 28:50], I liked it, it was interesting to me”. While mentioning the resource, Peter performed gesture #3MAL9.3 (Figure 54), quite similar to gesture #3UNI, where he moves his two hands apart, one to each side.

Figure 8.42: Gesture #3MAL9.3 made by Peter.



Source: Author (2024).

Figure 8.43: Gesture #3MAL9.4 made by Peter.



Source: Author (2024).

Therefore, considering the similarities of the depictive gestures, there is a possibility that the student was still developing a mental pictorial representation of the “fabric of the universe”, as mentioned by him, through interaction with the external psychophysical resource of the rubber-sheet model analogy.

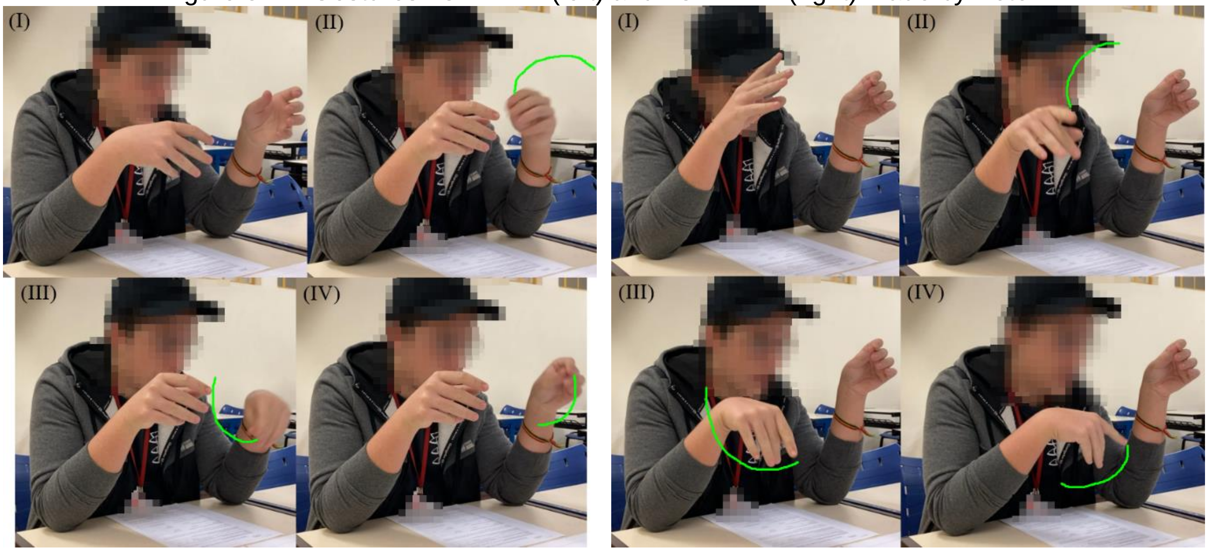
8.2.2 Gravitational Time Dilation

Regarding gravitational time dilation, Peter also presented an understanding still in development. For question 7, where one astronaut goes down to Earth while the other waits in space station, Peter answered in the pre-test that it would take “little less than 8 h” and in the post-test “also 8 h”. When explaining his response, Peter demonstrated not distinguishing between time dilation due to velocity of the reference frame, as described by SR, and gravitational time dilation:

> [I thought that] on Earth you don’t move, it’s a very small velocity, it’s just the Earth’s velocity as it’s moving [#VEL1.1 15:16], while the astronaut in the space station is moving at a very high velocity around the Earth [#VEL1.2 15:23]. Therefore, it would take a little less [time] for him. [...] [In the post-test] I thought it would have to be a greater velocity, even though the space station is very fast, it doesn’t have a velocity close to light speed, so there wouldn’t be any difference.

A students’ tendency to confuse time dilation by the speed and by mass was already observed in other studies (Gousopoulos; Kapotis; Kalkanis, 2016). Observing Peter’s explanation, he focused on the time dilation caused by the velocity of a reference frame, as evidenced by his depictive gestures #3VEL1.1 and #3VEL1.2 (Figure 8.44), where he moves each hand indicating the movement of the space station around the Earth. Possibly, the student does not recognize the Earth as a comparatively massive object that can cause changes in spacetime, even if minimal.

Figure 8.44: Gestures #3VEL1.1 (left) and #3VEL1.2 (right) made by Peter.



Source: Author (2024).

However, regarding question 8 of the test, which presents a similar situation, but around a black hole, the student presented a more coherent explanation about gravitational time dilation:

> [...] The black hole will dilate; it will dilate time. For those who are farther away, for those who are farther from the black hole more time will pass, and for those who are closer... I don't know exactly, but it's as if those who are closer [#3DIL2 17:31] to the black hole have passed some time, and for those who are farther away, the time [interval] will be bigger. [...] because it [black hole] is massive, it has more influence over time.

The student correctly stated that time would be slowed down near the black hole. During his explanation, Peter performed gesture #3DIL2 (Figure 8.45), where he initially moves his left hand away, representing someone near the black hole, then positions his hands in front of him, representing the black hole, and finally moves his right hand away, indicating someone farther away from the black hole.

Figure 8.45: Gesture #3DIL2 made by Peter.

Source: Author (2024).

Furthermore, Peter stated that the black hole's influence over time was due to "it being massive". This reinforces that the student possibly understands the influence of mass on time, but without recognizing the Earth as a comparatively massive object.

Peter’s difficulty is in accordance to Kersting; Henriksen; Bøe e Angell (2018) claims, that highlight the difficulty students face to apply Einstein ideas to their daily lives.

Finally, it is possible to observe that the influence of social mediation in Peter’s reasoning process, as he mentioned recording of “the formulas we used to calculate this type of question”. As all the exercises during the classes were developed in pairs, possibly this interaction with a classmate helped Peter to process information and start developing *drivers* which helped him to understand partially gravitational time dilation.

8.3 LIMITED UNDERSTANDING AND INCONSISTENT MENTAL REPRESENTATIONS

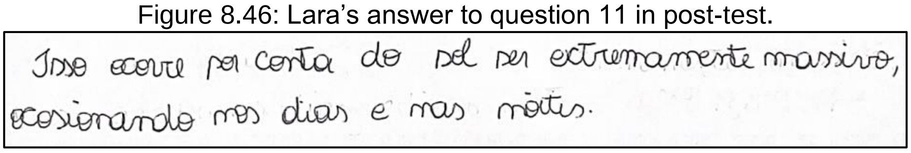
Some students demonstrated a limited understanding of General Relativity, particularly in relation to the concept of curved spacetime. These students were unable to provide correct answers to many questions and, in general, did not explain their responses in a coherent manner, or failed to provide any explanation at all. Among these students are Lara and Lisa, who will be presented here.

8.3.1 Curved Spacetime

The students in this group presented some common misconceptions. Among them, Lara and Lisa demonstrated a limited understanding of curved spacetime and were unable to answer correctly to many questions or explain them coherently. Furthermore, these students appeared not to understand the rubber-sheet analogy correctly and did not present solid mental representations.

In their answers to question 11 of the test (regarding the Earth’s orbit), some misconceptions can be observed. Lara confused the Earth’s translational and rotational movements, relating its orbit to the cause of days and nights: “This happens because the Sun is extremely massive, causing the days and nights”. Although Lara mentioned the Sun’s mass in her response and during the interview, it does not seem that she understood its influence on the Earth’s movement, that is, the curvature of space-time.

Figure 8.46: Lara’s answer to question 11 in post-test.

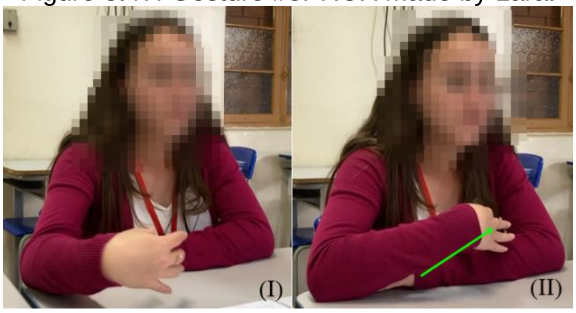

Isso ocorre por conta do sol ser extremamente massivo, ocasionando nos dias e nas noites.

Source: Author (2024).

> [...] I thought about the mass, and then I remembered, like, also the same situation of a black hole, when you approach [#3PROX 23:04] time passes more quickly, so I imagined that the Earth's movement [#3ORB3 23:09] would be the same situation.

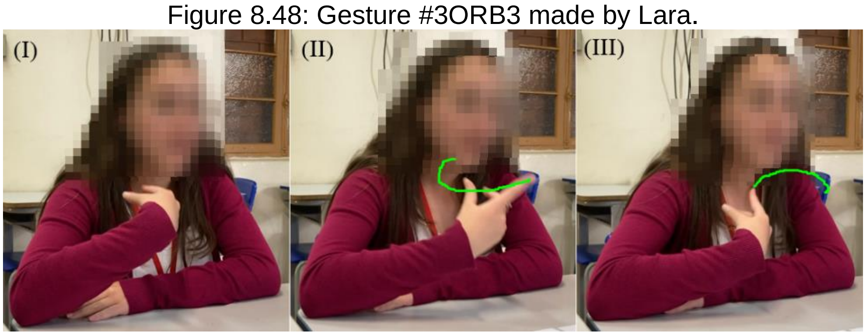
Although Lara presented a mental image of the Earth's orbit, represented by gesture #3ORB3 (Figure 8.48), where she moves her right index finger in a anticlockwise direction, she did not relate this to the curvature of spacetime. Additionally, she mentioned changes in time near a black hole, but did not establish a direct relationship between the Earth's orbit or the Sun's mass, as indicated by her speech and gesture #3PROX (Figure 8.47), where she moves her right hand towards herself, illustrating proximity to a massive body.

Figure 8.47: Gesture #3PROX made by Lara.



Source: Author (2024).

Figure 8.48: Gesture #3ORB3 made by Lara.



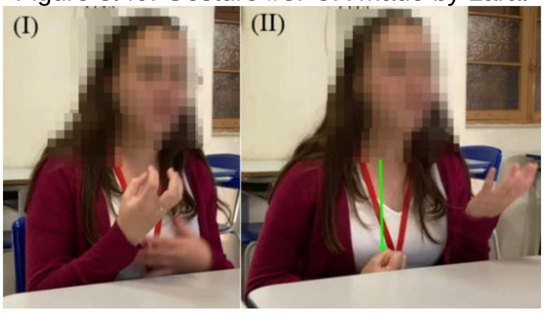
Source: Author (2024).

At another point, Lara mentioned the physical rubber-sheet model used in class:

> [...] I also remember now that experiment we did with some [#3PUX 23:56], that space pulls, that we put [#3MAL6.1 24:00] the fabric and threw the things [marbles]. That space pulls, and so there is a movement [#3MAL6.2 24:04] around. As if the Earth were moving, for example.

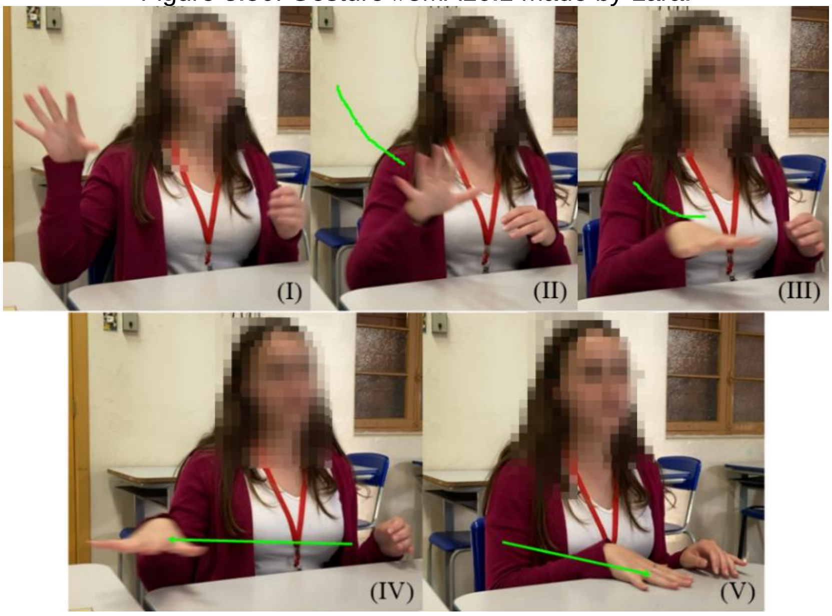
Despite Lara mentioning the rubber-sheet model, it appears that the student did not understand the meaning of the analogy. Therefore, she was unable to build mental representations (*drivers*) from her interaction with this external psychophysical resource. Lara associated the model with a force that "pulls", as indicated by her speech and gesture #3PUX (Figure 8.49), where she moves her right hand downwards, closing it, similar to #3PROX that she performed earlier.

Figure 8.49: Gesture #3PUX made by Lara.



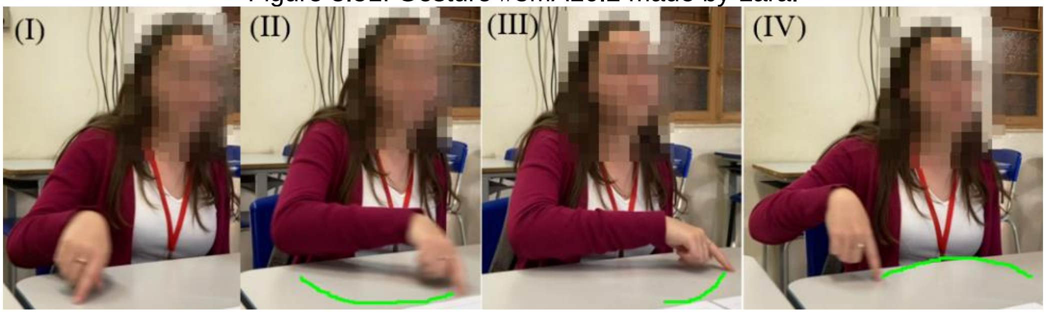
Source: Author (2024).

Figure 8.50: Gesture #3MAL6.1 made by Lara.



Source: Author (2024).

Figure 8.51: Gesture #3MAL6.2 made by Lara.



Source: Author (2024).

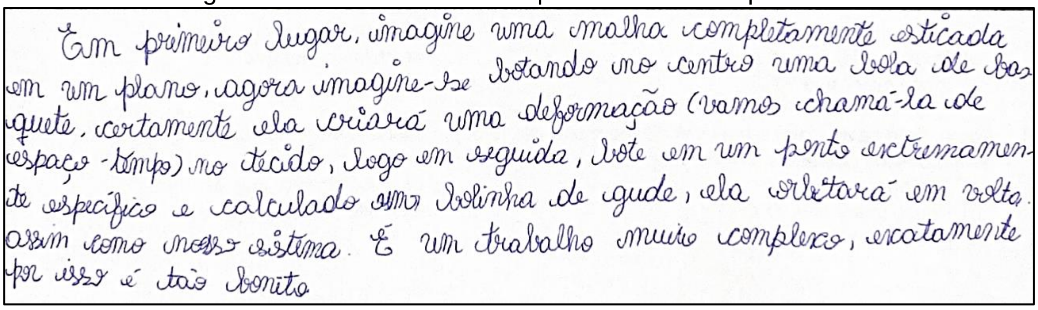
Lara seems to have related this force to space itself, rather than the curvature of spacetime caused by massive objects, as she mentioned “the space pulls”. This misconception may have been influenced by the rubber-sheet model, as it relies on the Earth’s gravity to work, possibly inducing students to think that objects are pulled by space, as in this case, pulled by the fabric. This is one of the limitations of this model presented in the literature (Postiglione; De Angelis, 2021b).

Furthermore, as indicated by her speech, “movement around, as the Earth would move”, and gestures #3MAL6.1 (Figure 8.50), moving her right hand in a curved motion downwards and then horizontally to the left, and #3MAL6.2 (Figure 8.51), moving her right index finger in an orbit over the table in a anticlockwise direction, Lara only internalized the movement of the marbles through the interaction with the psychophysical resource. She did not demonstrate the ability to assimilate the cause of this movement illustrated by the rubber-sheet analogy, the deformation of spacetime caused by masses.

Similarly, although the student Lisa also associated the rubber-sheet analogy with the Earth’s orbit, she did not demonstrate an understanding of the meaning behind the analogy, as she answered:

> Imagine a stretched rubber-sheet in a plane. Now, imagine that you put a basketball in the middle of it; it’ll create a deformation in the fabric (let’s call it spacetime). Thereafter, put [throw] a marble in a specific and exactly calculated point; it’ll orbit around the ball like our [Solar] system. It’s a very complex work, this is why it’s so beautiful.

Figure 8.52: Lisa’s answer to question 11 of the post-test.


Em primeiro lugar, imagine uma malha completamente esticada
em um plano, agora imagine-se botando no centro uma bola de bas-
quete, certamente ela criará uma deformação (vamos chamá-la de
espaço-tempo) no tecido, logo em seguida, bote em um ponto extremamen-
te específico e calculado uma bolinha de gude, ela orbitará em volta
assim como nosso sistema. É um trabalho muito complexo, exatamente
por isso é tão bonito

Source: Author (2024).

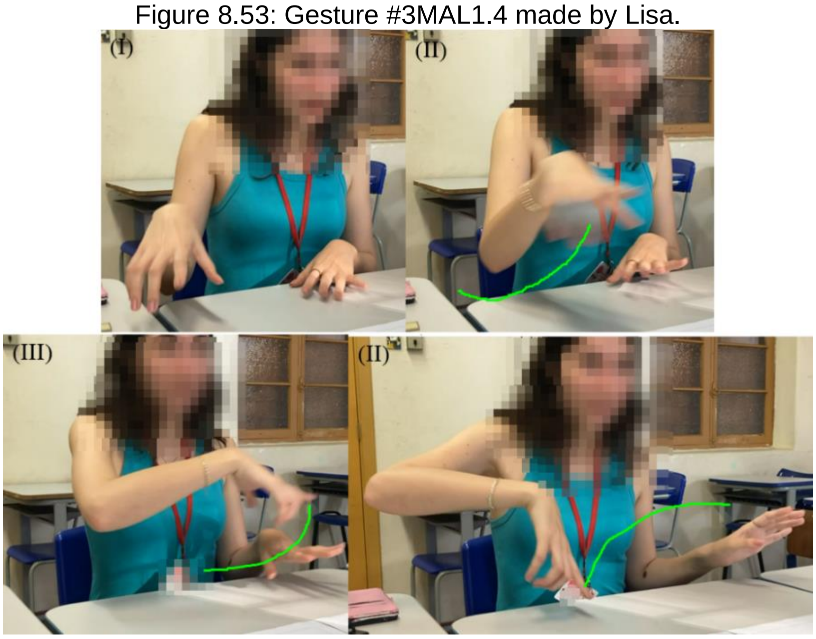
Lisa’s “rote learning” can also be observed during the interview. To explain her answer, Lisa simply described the rubber-sheet analogy, stating “I put exactly what I’ve learned”, suggesting that she only reproduced the information she obtained during the lessons for her answer to question 11:

> [...] I learned with the rubber-sheet, that they [marbles] stayed spinning and spinning [#3MAL1.4 2x 39:02]. I thought I’d answer like this, it’s possible to prove it. [...] I put exactly what I’ve learned in that class of the rubber-sheet, exactly what you said that it must have an exactly well-calculated distance, or it [movement] will be wrong.

It can be observed that Lisa focused on the movement of the marbles observed with the model, rather than the reason for this movement illustrated by the analogy.

This point can be observed in her speech when she explained that the marbles must be thrown in a precisely calculated way for the correct movement to occur and performed the gesture #3MAL1.4 (Figure 8.53), moving her right index finger in an orbit in a anticlockwise direction, which illustrates this movement.

Figure 8.53: Gesture #3MAL1.4 made by Lisa.



Source: Author (2024).

These comments indicate that Lisa possesses only a mental representation of the movement of the marbles, but not of the cause of this movement, the deformation caused by them. Therefore, Lisa did not develop appropriate mental representations, *drivers*, through her interaction with this psychophysical external resource and, therefore, did not understand the information provided by it about the cause of the movement observed.

Further indications of these students' limited understanding can be observed in their explanations regarding questions 9 and 10 of the test. Both questions dealt with the distortion of space caused by a black hole: "9) Two points close to the Earth have between them a distance of 1 km. What would be the distance between these points if they were close to a black hole in an altitude of 500 km?". In question 10, the points would be 20 km from the black hole.

Lara related the situation proposed by the question to differences in time and did not explain how the changes in space would occur. Analysing her speech and depictive gestures, it is possible to observe that she does not have a developed mental representation, neither pictorial nor propositional, for the changes in space caused by a massive body. As Lara explained during the interview:

> [...] the time would run faster the closer you get [to the black hole]. [...] I imagine a person, the black hole, and this person approaching [the black hole] [#3DIS4.2 21:29] and time changing, in terms of distance and how it would change.

It is possible to observe that Lara only imagined a person approaching the black hole and time changing, as indicated by the gesture #3DIS4.2 (Figure 8.54) where she indicates the black hole with her left hand and then moves her right hand towards it. Additionally, she incorrectly considered that time would pass more quickly near the black hole, as she stated that “the time would run faster the closer” to the black hole.

Figure 8.54: Gesture #3DIS4.2 made by Lara.

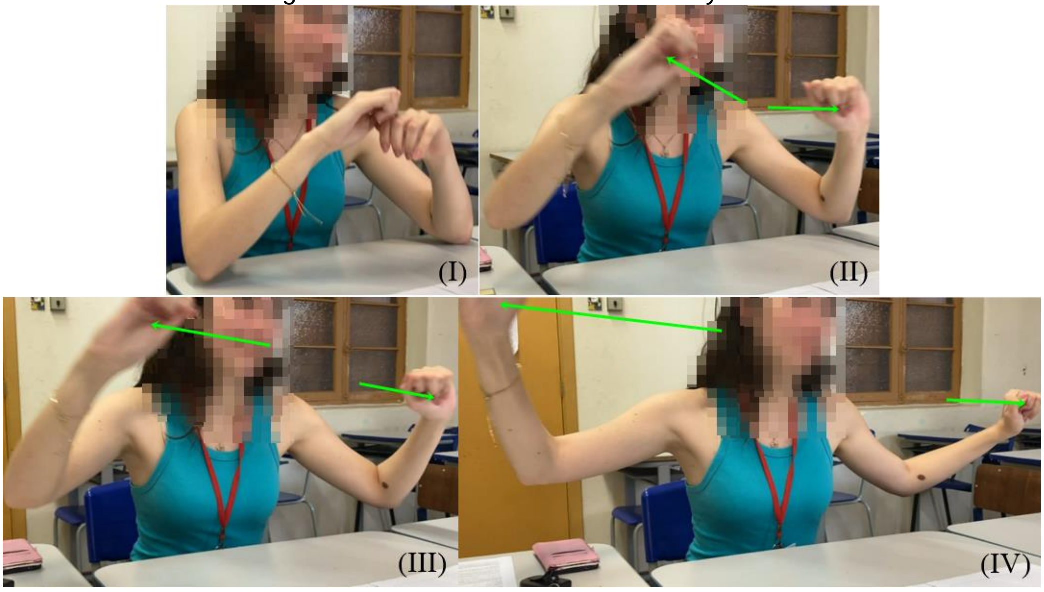
Source: Author (2024).

For the same questions, Lisa presented the idea of a force exerted by the massive object pulling other objects around it, rather than changes in space itself. Although the student correctly mentioned that the distance between the two points would increase, she associated this with the idea of forces acting in objects and the movement of these objects around the black hole:

> [...] for anything very close to a black hole, that spaghetti effect would happen [#3ESP2 33:11] of increase, stretch. I think that it [distance] would be much more than 1 [km], also it would have to make all that curve [#3MAL1.4 33:20] [...] [Further] I think that it [black hole] still having that force of trying to [#3SUG1.2 35:07 2x] suck.

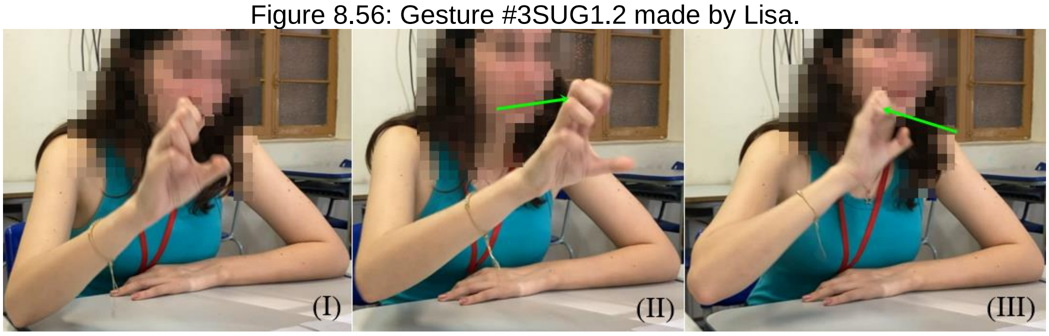
Observing Lisa’s gestures, it is possible to observe the ideas about the connection with forces. Firstly, representing something being stretched with the gesture #3ESP2 (Figure 8.55), where she places her two hands closed in front of her chest and then moves them horizontally, as if she was pulling an elastic band, then pulled with the gesture #3SUG1.2 (Figure 8.56), where she moves her right hand forward and backward, opening and closing it, as if she was grasping something.

Figure 8.55: Gesture #3ESP2 made by Lisa.



Source: Author (2024).

Figure 8.56: Gesture #3SUG1.2 made by Lisa.



Source: Author (2024).

As usual, students often struggle with this new interpretation of gravity (Steier; Kersting, 2019). Similarly to with the rubber-sheet model, here Lisa seems to have focused on the movement of objects caused by these forces, rather than the deformation of spacetime. This conception can be observed when she mentioned that objects “would have to make all that curve” performing again the gesture #3MAL1.4 (Figure 8.53). Lisa performed this gesture earlier when she mentioned the rubber-sheet model, which illustrates the movement of objects, reinforcing that she was not able to develop the appropriate *drivers* by the interaction with the psychophysical resource.

8.3.2 Gravitational Time Dilation

Considering Lara’s and Lisa’s images generated with AI, they did not demonstrate an understanding of GR. In her prompt, Lara connected the idea of relative time to the “outer space” but related it to “massive objects”. Prompt from Lara:

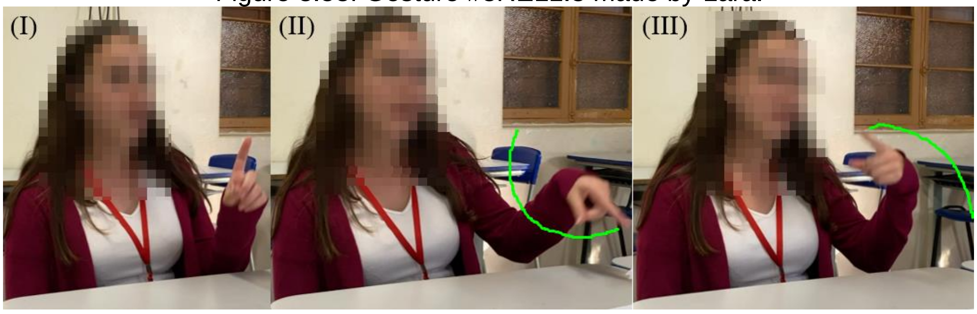
“When I think of the word ‘relativity’ I immediately imagine the space, the universe. I remember spaceships and how the time becomes relative when closer to a massive object”. During the interview, the student reinforced this idea:

> [...] I tried to think of something that is in the space, because space is my biggest reference that time is relative. [...] I started thinking with a clock working [#3REL1.3 25:47] at the same time. I remember that you talked about this in class, that we can imagine a clock running [#3REL1.3 25:51].

Figure 8.57: Lara’s final AI-generated image.

Source: Author (2024).

Figure 8.58: Gesture #3REL1.3 made by Lara.



Source: Author (2024).

Lara mentioned that space is her “biggest reference” for relative time, associating relativistic phenomena with outer space. She also mentioned a clock working, as discussed during the classes – social mediation – which can be observed in her image (Figure 8.57) and depictive gesture #3REL1.3 (Figure 8.58), a circular movement with her right index finger in a clockwise direction, illustrating the movement of the clock’s hands.

Although Lara did not have conceptions of curved spacetime, she presented an initial understanding of relative time, correctly associating it with massive objects. Possibly, this initial understanding was developed through social mediation, as she mentioned the explanations during the lessons. Similar ideas were observed when Lara explained her answers to questions 7 and 8 of the test, addressing the gravitational time dilation around the Earth and a black hole, respectively.

Lara associated the changes in time with mass, but with the idea that only large masses in "space" would cause effects on the passage of time:

> [...] Because I was always thinking about outer space, like, someone that stays at Earth and someone that goes to outer space. So, as there [outer space] it's close to a big mass. Whoever goes to space, the time changes, compared to those who stayed on Earth.

Here, Lara presented the same way of thinking she used to generate her image. As she mentioned earlier, she used "the space" as her reference for an image of "relativity". Now, dealing with gravitational time dilation, she associated again the phenomenon with "outer space". Therefore, Lara seemed not to recognize the Earth as comparatively a massive body, having difficulty to connect Einsteinian ideas to her daily life (Kersting; Henriksen; Bøe; Angell, 2018).

Considering Lisa's AI-generated image, it was also possible to observe that the concepts involving the theory were not clear for her. The resulting image was abstract, as Lisa was quite vague in her prompt: "Relativity is something difficult to be proven precisely due to distinct reference frames, relativity is the beauty of the uncertain, it is ambiguous".

Figure 8.59: Lisa's final AI-generated image.

Source: Author (2024).

Although the resulting figure was undefined (Figure 8.59), without apparent physical meaning, it met Lisa's expectations, as she mentioned that the lines in the image represented different movements "in a velocity that depends on who is watching". Therefore, the "reference frames" mentioned by Lisa bring the idea of relativity related to "different viewpoints", associating a reference frame with the presence of observers, a common error among students (Panse; Ramadas; Kumar, 1994).

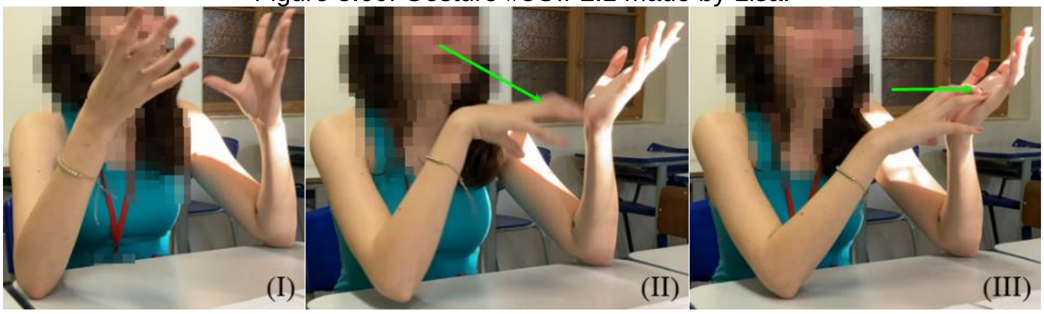
Discussing questions 7 and 8 of the test, in the same way as Lara, Lisa also associated the differences in time with “being outside the Earth”. Explaining her answers, Lisa mentioned that for those who go to space, time passes more slowly, while for someone on Earth “passes our time”, suggesting an idea of a “real” or “absolute” time:

> The one [astronaut] that stays at the spaceship, the time would pass slower for him. And the one that stayed at Earth, it would pass our time […] Because the time at outer space passes less […]

It is possible to observe in her speech that Lisa also connected time dilation to “outer space”, because there, as she mentioned, time would pass more slowly. So, like Lara, Lisa presented some difficulties to associate the Einsteinian ideas to her everyday life. Then, Lisa commented on the effects of massive objects:

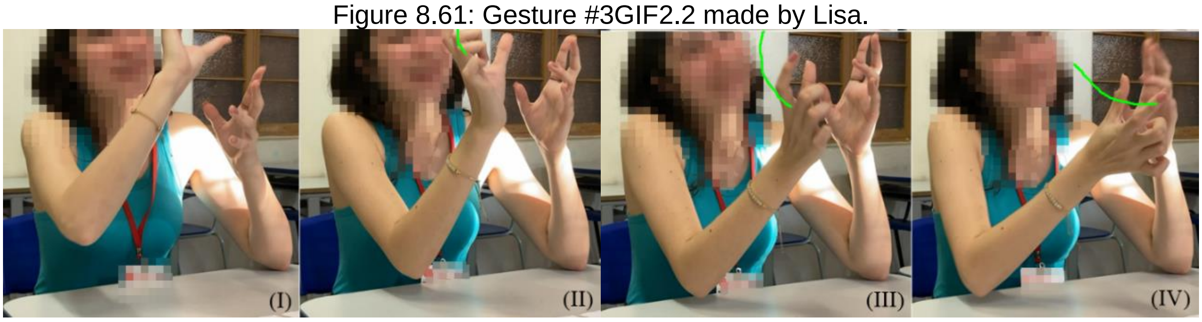
> […] We talked a lot about this during the classes, after we watched that Interstellar scene, that time really passes more slowly. And about the black holes, that closer [#3GIF2.1 28:18] to a black hole, to a massive object, time passes very slowly, like a clock very slowly [#3GIF2.2 28:24].

Figure 8.60: Gesture #3GIF2.1 made by Lisa.



Source: Author (2024).

Figure 8.61: Gesture #3GIF2.2 made by Lisa.



Source: Author (2024).

The student demonstrated an initial understanding of time dilation and a mental representation still in development for this phenomenon, as evidenced by the depictive gestures #3GIF2.1 (Figure 8.60), where she places her two hands and then moves her right hand towards the left, representing the approach of the black hole, and #3GIF2.2 (Figure 8.61), where she places her right hand in the shape of a "C" representing a clock and then rotates it, representing the clock hands moving more slowly.

These gestures and discourse indicate that Lisa imagined a clock being slowed down as someone approaches a black hole. However, although Lisa correctly associated the slowing down of time with the proximity of massive objects, she demonstrated not recognizing the Earth as a comparatively massive object.

This initial understanding of gravitational time dilation is likely associated with social mediation, as Lisa remembered the class discussions, and cultural mediation, through the scene from the movie *Interstellar*. By interacting with these external resources, Lisa began to develop her *drivers*, in the form of a mental representation, which helped her obtain this partial understanding of gravitational time dilation.

Possibly, both Lisa and Lara had only just begun to develop their mental representations for time dilation, which were not yet established. The social mediation, during the interaction with the classmates and teacher, might have helped them to process information about gravitational time dilation, as both students mentioned during the interview. They demonstrated relating the changes in time to massive objects, but without recognizing the Earth as one, presenting the idea that the changes would occur only in “outer space” and “outside the Earth”.

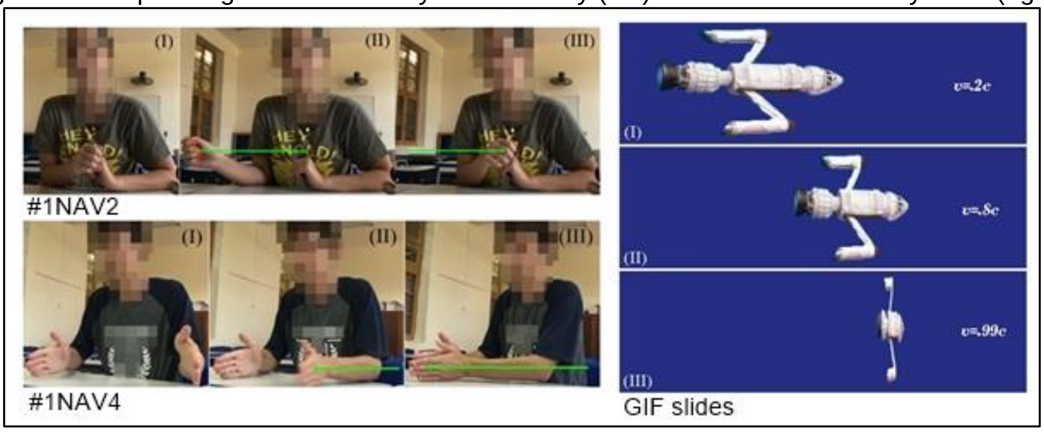
9 MENTAL REPRESENTATIONS AND RELATIVITY UNDERSTANDING

The obtained results suggest a strong connection between the consistency of the mental representations constructed by students and their understanding of the concepts these representations involve. Students with satisfactory comprehension presented clear and well-developed mental representations of the phenomena. These students demonstrated a general understanding of the situations they had worked on, correctly answering many of the test questions. On the other hand, students who presented inconsistent mental representations showed a limited understanding of relativistic phenomena.

In Cohort 1, students with satisfactory comprehension exhibited clear evidence of consistent mental representations for space contraction using their representations to solve different problems proposed. These mental representations were developed in the form of mental simulations and could be identified through depictive gestures and detailed descriptions on the interviews, when these students reported “imagining” or “seeing” the phenomenon happening (see chapter 6, section 6.1.2).

Both students with satisfactory comprehension, Finn and Lily, performed similar depictive gestures and mentioned remembering a GIF that was used in the slides of a spaceship increasing in speed and changing size (Figure 9.1). Therefore, the two students developed similar pictorial mental representations to address the same phenomenon, space contraction.

Figure 9.1: Depictive gestures made by Finn and Lily (left) and GIF mentioned by them (right).



Source: The author (2024).

These students also understood the progressive nature of relativistic effects in space according to the velocity of the reference frame, and accepted the validity of space contraction, rather than relating it to perception. Particularly, Finn was able to change between different reference frames, transposing the situation of the spaceship GIF to the ground, or the path of a moving truck in question 3 of the post-test (see chapter 6, section 6.1.2). However, Finn was the only student who demonstrated overcoming this common unilateral view of space contraction (Alstein; Krijtenburg-Lewerissa; Van Joolingen, 2021).

Regarding the phenomenon of space contraction, when travelling at high velocity, students with partial comprehension and those with limited comprehension presented similar results. These students did not demonstrate that they possess a consistent mental representation for the phenomenon, imagining a blur due to high velocity. Consequently, these students associated this phenomenon of space contraction with an inability to measure correctly the moving object, or even with the observer not being able to perceive the event.

Both students, Elle and Iris, with partial understanding (see chapter 6, section 6.2.2), and Eve and Nora, with limited understanding (see chapter 6, section 6.3.2), performed similar depictive gestures, indicating something very fast and difficult to visualize (Figure 9.2). These students also were unable to explain verbally the space contraction, indicating the absence of mental representations even in the propositional format – in other words, these students were not able to understand space contraction.

Figure 9.2: Iris, Elle, Eve and Nora's depictive gestures (top) and image showing a car at high speed (bottom).

Source: Author (2024).

Regarding time dilation, the development of mental representations also seemed to be necessary for understanding the phenomenon. In this aspect, students

Elle and Iris, with partial understanding, presented similar results to Finn and Lily, with satisfactory understanding. The four students did not present any evidence of pictorial mental representations. When explicitly asked about any mental image for time dilation, none of the students claimed to have one.

However, by analysing the verbal explanation used by these students, it is possible to identify evidence of their conceptual understanding. In this sense, it can be inferred that these students developed another type of mental representation for time dilation with logical sentences that helped them understand the phenomenon. The propositional representations that they developed established a relationship between the velocity of the reference frame and the passage of time, as shown by the quotes in Table 9.1.

Table 9.1: Students' interviews excerpts indicating their propositional mental representations of time dilation.

Finn	Lily	Elle	Iris
"it's a much higher speed, so it can decrease the time"	"as slower the speed, less is the difference"	"less than one year for the person who was travelling"	"time passes faster for those who are there, outside [the boat], not travelling"
Connection with exercises and explanations			

Source: Author (2024).

Instead of representing these relationships through images, these four students were able to express their understanding through simple sentences, indicating that they used propositional mental representations. As a result, they were able to correctly understand time dilation and its gradual effect with increasing velocity. These students also recognized time dilation as a tangible phenomenon and not just a perception of the time passage.

As mentioned earlier, none of the students seemed to be able to change between different reference frames for time dilation. All students explained the phenomenon from a stationary observer's perspective relative to the ground, presenting a unilateral view (Alstein; Krijtenburg-Lewerissa; Van Joolingen, 2021).

However, considering that these students had a brief contact with Special Relativity in the previous lessons and considering their level of instruction was the same for each member of the class, it was anticipated that they would present limitations, like the unilateral view, in their understanding. As observed in the literature review (see chapter 3, section 3.2.3), even university students present serious difficulties in acquiring a deeper understanding of SR (Yavaş; Kızılcık, 2016), so it can be stated that these students developed a satisfactory understanding of time dilation.

On the other hand, students with limited understanding did not present any evidence of structured mental representations for time dilation, in any format. Therefore, these students tended to relate time dilation to a psychological perception of the passage of time (Table 9.2).

Table 9.2: Eve's and Nora's interviews excerpts indicating the connection stablished with perception of time passage.

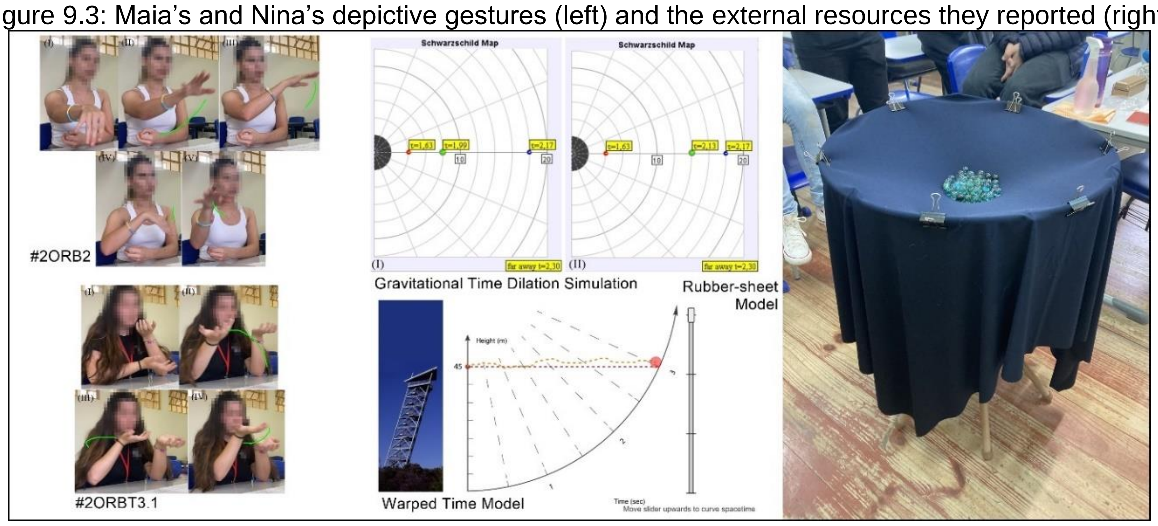
Eve	Nora
"whoever is inside the train cannot understanding it [time passage], because it's [train] going too fast"	"It [the train] is moving and the person [inside it] wouldn't realize that time was passing"
Perception of time passage	

Source: Author (2024).

Additionally, students with limited understanding frequently associated relativistic effects with only extremely high velocities, showing a lack of understanding of the role of the gamma factor, namely that in low velocities the effects exist, but minimally. These results are in line with the main difficulties identified when learning Special Relativity in secondary schools discussed by Hughes and Kersting (2021) and Kamphorst, Vollebregt, Savelsbergh and Van Joolingen (2023).

Similar results were observed among students in Cohort 2 and Cohort 3 for understanding concepts of General Relativity. In Cohort 2 satisfactory understanding group Maia and Nina demonstrated clear pictorial mental representations for curved spacetime. By analysing these students' depictive gestures and reports was possible to identify the mental representations (Figure 9.3).

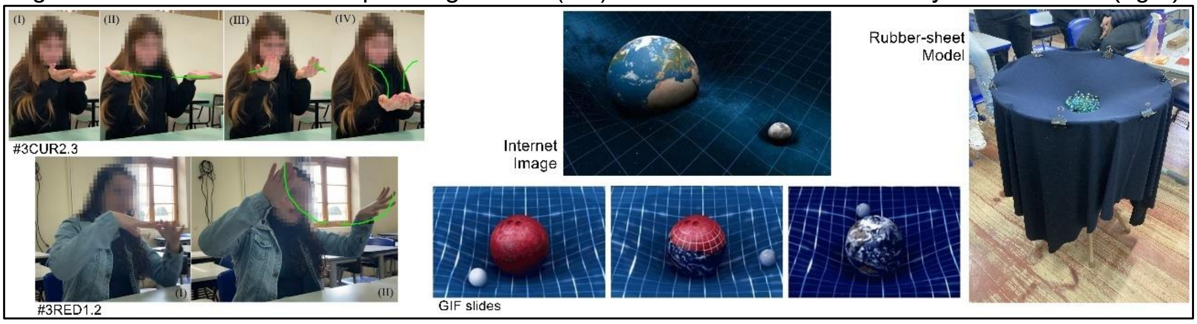
Figure 9.3: Maia's and Nina's depictive gestures (left) and the external resources they reported (right).



Source: Author (2024).

In Cohort 3, Sam and Ana, with satisfactory understanding, also demonstrated possessing well-defined pictorial mental representations of curved spacetime. The mental representations of these students could be identified through their depictive gestures and reports during their interviews (Figure 9.4).

Figure 9.4: Sam and Ana's depictive gestures (left) and external resources they remembered (right).



Source: Author (2024).

These mental representations were quite consolidated, as all students used them to solve different problems in different situations, like the Earth's orbit and light bending. This could be observed when Maia, Nina, Sam and Ana repeated their depictive gestures or performed similar gestures, describing the same type of mental images (see chapter 7, section 7.1.1 and chapter 8, section 8.1.1).

With partial understanding, Leia presented a pictorial mental representation for curved spacetime still in development, as she was not able to explain the "gravitational fabric" mentioned by her (see chapter 7, section 7.2.1). In Cohort 3, Peter also presented a pictorial representation of curved spacetime, this representation was not yet consolidated in that he demonstrated using this representation only in one situation during the interview, not applying it to explain different phenomena (see chapter 8, section 8.2.1).

Finally, the students with limited understanding did not demonstrate concrete conceptions of curved spacetime. In Cohort 2, Mike explained the Earth's movement by the Sun's force of attraction, he even mentioned the "gravitational force" in his answers. In Cohort 3, Lara and Lisa associated the effects of a black hole, for example, with forces that "pull" or "suck". These conceptions could be observed in the students' discourses and were evidenced by their depictive gestures (Figure 9.5).

Additionally, despite both Lara and Lisa during the interview mentioned remembering the rubber-sheet model, they apparently internalized only the movement performed by the marbles, and not the curvature demonstrated by the model.

Therefore, they could not understand the meaning of the analogy, and did not use the curvature of spacetime to explain the phenomena addressed.

Figure 9.5: Mike's, Lara's and Lisa's depictive gestures indicating the connection with force conceptions.

Source: Author (2024).

Regarding gravitational time dilation, the students Maia (Cohort 2) and Ana (Cohort 3) demonstrated a more consolidated understanding. Both students demonstrated understanding that massive objects slow down the passage of time around them, as well as the relationship between the amount of mass and the influence caused by time in different locations (Table 9.3).

Both Maia and Ana demonstrated a good understanding, without resorting to the use of images, but rather they created structured sentences to explain their understanding. Therefore, it can be inferred that these students developed propositional mental representations for the phenomenon of gravitational time dilation.

Table 9.3: Maia's and Ana's interviews excerpts where they related the time passage with mass.

Maia	Ana
"I remembered of the movie we discussed […] The mass of the planet changed how time passed" Question 7 – "[The astronaut] is closer to that mass, then time would pass more slowly [for him]" Question 8 – "near the black hole, the distance, the differences, everything gets amplified"	"we studied that part about mass, about the more massive objects" Question 7 - "the person there [space station], would pass more time than the person who is there on Earth" Question 8 – "for him [astronaut who stayed on the spaceship], it would take longer. [...]I remembered when the teacher talked about black holes being very massive"
Connection between mass and time passage	

Source: Author (2024).

Other students with satisfactory understanding, like Nina (Cohort 2) and Sam (Cohort 3), although having a good understanding of curved spacetime, presented more limited ideas about gravitational time dilation. Their conceptions about this phenomenon were similar to those of Peter (Cohort 3), a student designated as having partial understanding of time dilation.

These three students, while explaining question 7, which involved an astronaut near the Earth, demonstrated not recognizing Earth as a massive object. In this question, some of these students ended up relating the situation to time dilation due to the velocity of the reference frame, treated by the Special Relativity. However, for question 8, when dealing with a black hole, all three students recognized its effects on the passage of time. They mentioned that due to the object being massive, it would slow down the time around it (Table 9.4).

Table 9.4: Nina's, Sam's and Peter's interviews excerpts indicating they recognize the massive objects effects on time passage, although they do not recognize Earth as one.

Nina	Sam	Peter
"The astronaut who stayed there [space station], it passed less [than 8 h] […] Maybe [it would be] also 8 h. Because there's nothing [massive] there to interfere with the time"	"As in the brothers' situation, in the brothers' question, time passed much more slowly for the brother who stayed in space. So, I used this idea to answer this question"	"I thought it would have to be a greater velocity [than at Earth], even though the space station is very fast, it doesn't have a velocity close to light speed, so there wouldn't be any difference"
Do not recognize Earth as a massive object		
"a black hole is a very massive object, due to that, time passes very differently there [near it] […] for that person who stayed here [spaceship], I imagined it would pass much more time […] I remember [the movie] they both had to compare the time, on her watch and on his watch [#2REL2.1 30:41], to see how different it would be [#2REL2.2 30:44 2x]".	"[In the simulation] Here [#3DTG4.2 3x 15:57] and here there wasn't much difference between one point and the other, because it's close to the black hole [#3DTG4.3 16:00], which is massive. Now, here [#3DTG4.4 16:02] passes a different time, which is a longer time than here [#3DTG4.5 16:05]"	"The black hole will dilate; it will dilate time. For those who are farther away, for those who are farther from the black hole more time will pass [...] because it [black hole] is massive, it has more influence over time".
Recognize the influence of mass deaccelerating the passage of time.		

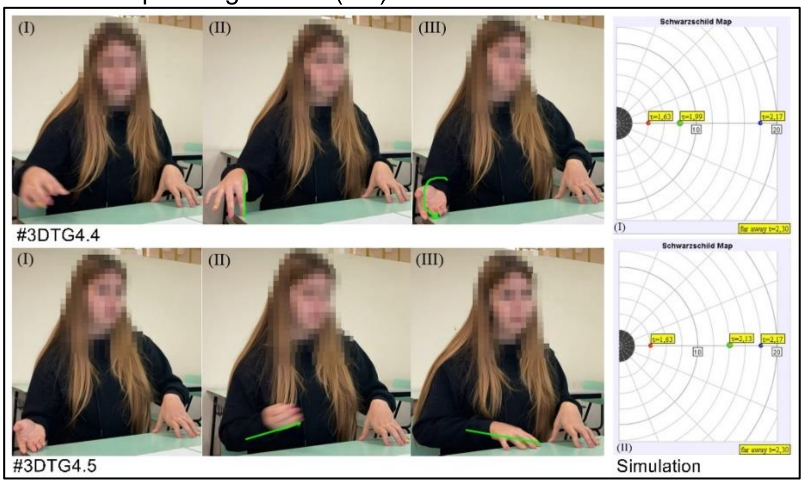
Source: Author (2024).

Nina and Sam even presented indications of developing pictorial mental representations. Nina mentioned remembering a scene from the Interstellar movie of two watches and performed depictive gestures representing these watches and their hands' movements (Figure 9.6). Sam claimed to remember a computer simulation used during classes; she performed depictive gestures and recalled what she visualized in the simulation (Figure 9.7). These are indications that these students internalized part of what was observed, in the movie or in the simulation, in the form of pictorial mental representations.

Figure 9.6: Nina's depictive gestures (left) and the movie scene she remembered (right).

Source: Author (2024).

Figure 9.7: Sam's depictive gestures (left) and the simulation she remembered (right).



Source: Author (2024).

The student Leia (Cohort 2), even with an initial understanding of curved spacetime, presented a limited understanding of gravitational time dilation. She did not demonstrate to relate the mass objects to time passage, only the distance. However, even without connecting the mass to time passage directly, Leia recognized that a black hole would cause more effects in time (Table 9.5).

The students Lara and Lisa (Cohort 3), who did not present comprehension about curved spacetime, demonstrated an initial understanding of time dilation. Their results for this phenomenon were similar to those of Sam and Peter, although they erroneously associated that "in space, time is different", when in class the activities for this concept showed that the amount of mass would cause effects on time. However, both Lara and Lise, when dealing with a black hole, mentioned the influence of mass on the passage of time (Table 9.6).

Table 9.5: Leia's interview excerpts indicating her limited comprehension about time dilation.

Leia
"in the simulations you showed, the time interval was very different from one [point] to another. Therefore, I put much more than 8 h" "The time and the distance around the black hole are different from what they would be on Earth. [...] As you get closer to the black hole […] time will pass differently.
Connection between distance and time passage

Source: Author (2024).

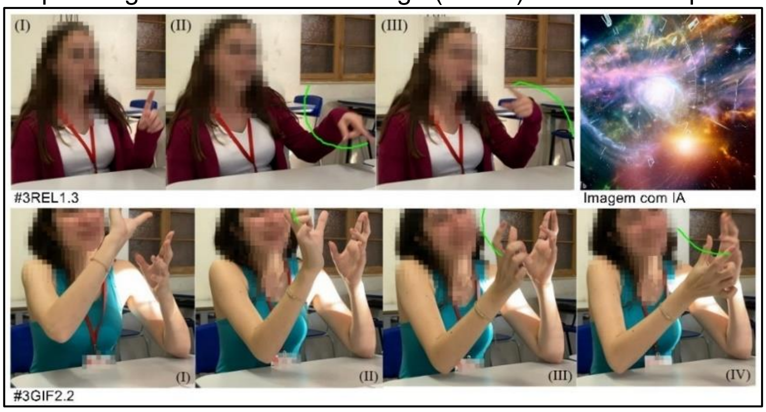
Table 9.6: Lara's and Lisa's interviews excerpts indicating their initial comprehension of time dilation.

Lara	Lisa
"space is my biggest reference that time is relative. [...] I started thinking with a clock working [#3REL1.3 25:47] [...] I was thinking about outer space, someone that stays at Earth and someone that goes to outer space. As there [outer space] it's close to a big mass, whoever goes to space, time changes"	"The one [astronaut] that stays at the spaceship, the time would pass slower for him. And the one that stayed at Earth, it would pass our time […] Because the time at outer space passes less […] closer [#3GIF2.1 28:18] to a black hole, to a massive object, time passes very slowly, like a clock very slowly [#3GIF2.2 28:24]"
Do not recognize Earth as a massive object, time is different in outer space	

Source: Author (2024).

Interestingly, the two students also mentioned imagining a clock running slowly for the phenomenon of time dilation. Additionally, Lara and Lisa also performed depictive gestures (Figure 9.8) indicating the presence of a pictorial mental representation, even if still in development stage. Lara even used this idea to generate her image with AI.

Figure 9.8: Lara's depictive gesture with her AI image (above) and Lisa's depictive gesture (bellow).



Source: Author (2024).

Surprisingly, for the student Mike (Cohort 2), even though he did not present any conceptions of curved spacetime, he presented a good understanding of gravitational time dilation. He correctly associated the objects' amount of mass to their influence over time, as well as the distance from these objects (Table 9.7).

Table 9.7: Mike’s interview excerpts indicating his good comprehension about time dilation.

Mike
“I imagined the situation from the movie [Interstellar][…] I thought it was, the more massive the object, the slower time will pass. […] for the one on the planet near the massive object, time is slower”.
Connection between mass and time passage

Source: Author (2024).

The observed results allow the researcher to establish a direct connection between the development of consistent mental representations and the understanding of phenomena related to them. It was possible to observe a certain tendency to use pictorial representations when dealing with concepts of space contraction and curved spacetime, and propositional representations when dealing with time dilation. Evidently, some students resorted to the use of images for the representation of time, such as Lara and Lisa. However, the only consolidated representations observed that result in a satisfactory understanding of time dilation were in the propositional format.

A possible reason for the difference between the mental representations presented by students for different phenomena is the nature of these concepts. The idea of “space” or even “curvature” is more tangible and concrete, possibly favouring the construction of pictorial mental representations. On the other hand, the concept of “time” is more abstract, and is less related to images directly. Therefore, the use of propositional mental representations may have been facilitated.

Another relationship that can be established is to measure how much each format of representation demands of the human brain capacity. According to Kosslyn, Thompson and Ganis (2006), mental representations in the form of images or pictorial representations require a higher capacity for cerebral storage than propositions. Therefore, it may be possible that for a complete understanding of time dilation, it is not necessary to use pictorial representations, only using a propositional format which enables represent the key aspects of it.

These results suggest that the development of both formats of mental representations, pictorial and propositional, significantly contributes to conceptual understanding about complex scientific topics such as Relativity Theory. This observations and finding aligns with the ideas of Kosslyn, Thompson and Ganis (2006), that different formats of mental representations highlight different information, and therefore can be used in different situations.

10 FINAL CONSIDERATIONS

In this doctoral research, the conceptions of high school students regarding their understanding of Relativity Theory were analysed, focusing on the mental representations developed by them during their learning process. The investigation began by examining mental representations related to the Special Relativity Theory (SRT) in Cohort 1. These results guided the planning for Cohorts 2 and 3, which addressed the General Relativity Theory of (GRT).

A detailed review of the literature showed that relativity is a challenging topic for students, primarily due to the invisibility of relativistic phenomena in their daily experiences (Hughes; Kersting, 2021; Kamphorst; Vollebregt; Savelsbergh; Van Joolingen, 2023). Therefore, the initial hypothesis of this research was that the development of mental representations by students, whether pictorial or propositional (Kosslyn; Thompson; Ganis, 2006), about relativistic phenomena could aid students in understanding the Relativity Theory.

To assist students in the development of these mental representations, teaching activities were planned using different levels of external mediation according to the Cognitive Mediation Networks Theory (Souza; Da Silva; Da Silva; Roazzi *et al.*, 2012; Souza; Serrano; Roazzi, 2024) were planned. With these activities planned, the following specific objectives of the thesis were achieved:

- Develop a teaching sequence addressing since Galilean Relativity to the General Relativity Theory;
- Discuss didactically the flaws in Galilean Relativity, demonstrating the need for the development of the Relativity Theory;
- Present the historical facts that led to the development of the Relativity Theory;
- Present conceptual models for Special Relativity within the different levels of mediation of the CMNT;
- Present conceptual models for General Relativity within the different levels of mediation of the CMNT.

The results and discussion presented in the previous chapters (see chapters 6, 7, 8 and 9), highlighting the strong connection between the characteristics of students' mental representations and their conceptual understanding allowed to achieve the last two specific objectives:

- Identify and analyse the students' conceptions for understanding General Relativity;
- Identify and analyse the mental representations constructed by students for understanding General Relativity, whether propositional or pictorial.

In this context, the main research question addressed in this thesis was: "How do students' mental representations and conceptions of Relativity Theory develop?". To answer this question, the first section of this chapter discusses the influence of external mediation resources on the process of developing mental representations. Considering the connection between this mental representations and conceptual understanding, this understanding was also influenced by the external mediation resources.

The second section of this chapter discuss to role of the GenAI tools used in this research in the investigation of students' conceptual understanding. The third section presents the limitations of this research, as well as discuss some suggestions for future directions based on the observed results. Finally, the main implications of the obtained results for research in physics education and teaching practice are presented in the fourth section.

10.1 EXTERNAL RESOURCES' INFLUENCE ON MENTAL REPRESENTATIONS

Through the results obtained, it is possible to observe that external resources, at different levels of mediation, had an influence on the mental representations developed by the students. This result aligns with the Cognitive Mediation Networks Theory (Souza; Da Silva; Da Silva; Roazzi *et al.*, 2012), as the theory suggests that mental representations are the *drivers* that are developed through interaction with these external mediations. This is comparative to a computer downloaded *drivers* when interacting with external devices like a printer.

Regarding Cohort 1, students with satisfactory understanding demonstrated developing pictorial mental representations for space contraction, with impressive similarities for the two of them: a spaceship or rocket changing size as it increased its velocity. This mental simulation reflects a GIF displayed in the slide presentation, which students explicitly referred. Therefore, it is possible to observe the influence of hypercultural mediation in the construction of these mental representations, representing a phenomenon absent from students' daily experiences.

Regarding time dilation, it was also possible to note the influence of specific levels of mediation on the construction of students' mental representations. For this phenomenon, students with satisfactory understanding and partial understanding presented similar results and referred to the same external mediation level, namely, social mediation. These students mentioned that they remembered exercises and discussions during classes with their peers and the teacher. By discussing situations involving time dilation during classes, through social interaction, students were able to construct their own mental representations. These students' reports highlight the role of social mediation in enabling the development of propositional mental representations.

On the other hand, for space contraction, students with partial understanding presented similar results to those with limited understanding. The mental images described by these students consisted of "blurs", indicating inconsistent mental representations. Probably, the students developed these blurred images from daily experiences of objects moving at high speeds – psychophysical mediation. However, space contraction is a phenomenon that cannot be observed through daily experiences, so these students were not able to use the other levels of mediation, such as hypercultural, to establish the relationship between the velocity and space measurements.

Finally, students with limited understanding were unable to develop adequate mental representations even for time dilation. For this phenomenon, similarly to their description of space contraction, these students also linked their understanding to their daily experiences, psychophysical mediation, associating the phenomenon with the psychological perception of time passage.

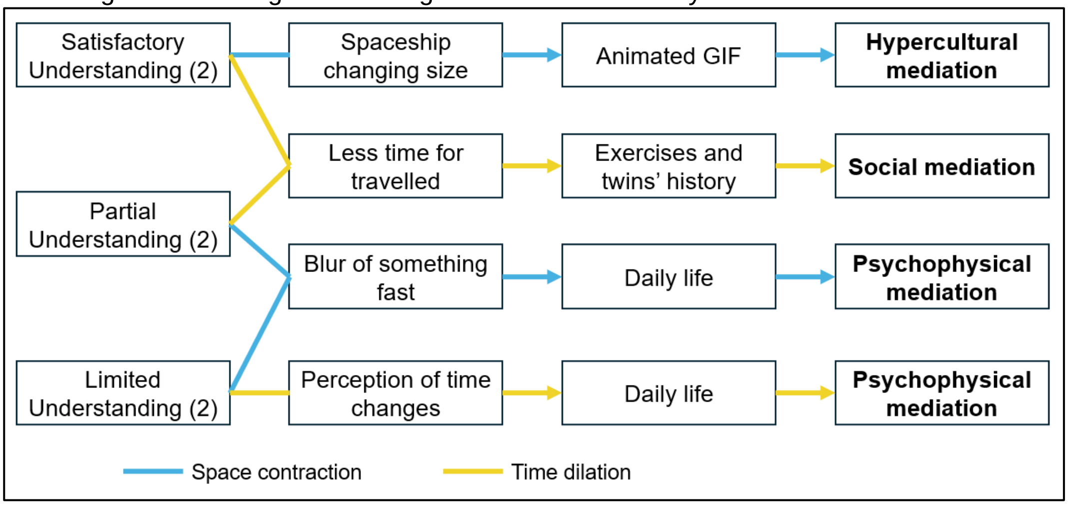
It is possible to observe, therefore, that these students used ideas from their everyday life to explain relativistic phenomena. In this sense, their daily experiences became obstacles to the development of mental representations and conceptual understanding. This is a common difficulty among students when learning complex topics such as Special Relativity; Alstein, Krijtenburg-Lewerissa and Van Joolingen (2023) observed many students tended to use classical ideas to explain relativistic situations in a simulation, and Kamphorst, Vollebregt, Savelsbergh and Van Joolingen (2023) identified some students who had difficulties in accept new models to explain the phenomenon studied.

One possible explanation for this difficulty, within the framework of the Cognitive Mediation Networks Theory (CMNT), is that these students' mediation profiles might be tied to specific levels of external mediation. As students primarily rely on psychophysical and social mediations grounded in their everyday experiences, they may struggle to engage with the more abstract and counterintuitive concepts of relativity, which often require hypercultural tools, like simulations and mathematical models, to be represented. This limitation in using diverse levels of mediation can hinder the development of consistent mental representations, necessary for conceptual understanding (Anjos; Serrano, 2024).

Other perspective suggests that students' conceptual profiles may be a limiting factor (De Souza; Serrano, 2021; Mortimer, 1995). In line with Mortimer's conceptual profiles, the evolution of students' understanding of a scientific concept, such as time dilation, might be constrained by their current stage of conceptual development. For instance, a student anchored in a Newtonian understanding of absolute time might struggle to grasp the relativistic notion of time as a relative dimension. As relativistic phenomena are not directly observable in everyday life (Yavaş; Kızılcık, 2016), this struggle might be further intensified, making it challenging for students to construct consistent mental representations grounded in daily experiences. Further research is needed to investigate the interplay between the level of mediation and the conceptual profile in shaping students' understanding of relativity.

Figure 10.1 presents a diagram summarizing the external resources and respective levels of mediation used by students in Cohort 1 for constructing their mental representations of space contraction and time dilation.

Figure 10.1: Diagram showing the mediation used by students from Cohort 1.



Source: Author (2024).

The results obtained with Cohort 2 and Cohort 3 also indicate a connection between students' mental representations and the external mediations with they have interacted. For example, Maia and Nina were able to connect the ideas of curvature they observed in the rubber-sheet model and simulations to their mental representations for the cause for Earth's orbit. Both students performed depictive gestures representing the Earth's movement with their hands going up and down, indicating the curved path for the planet (see Figure 7.2 and Figure 7.11). Both Maia and Nina used the interaction with psychophysical and hypercultural levels of mediation to develop their *drivers*, that is, pictorial mental representations of curved spacetime.

Leia also presented indicators of a mental representation for curved spacetime, as observed in her speech and depictive gestures (see chapter 7, section 7.2.1). Nonetheless, Leia's mental representation is still in development, as she was not able to explain how the "gravitational fabric" would cause the Earth's movement. During the interview, Leia mentioned recalling of a psychophysical resource, the physical rubber-sheet model. Possibly, her *drivers* from the interaction with this external resource were still not consolidated.

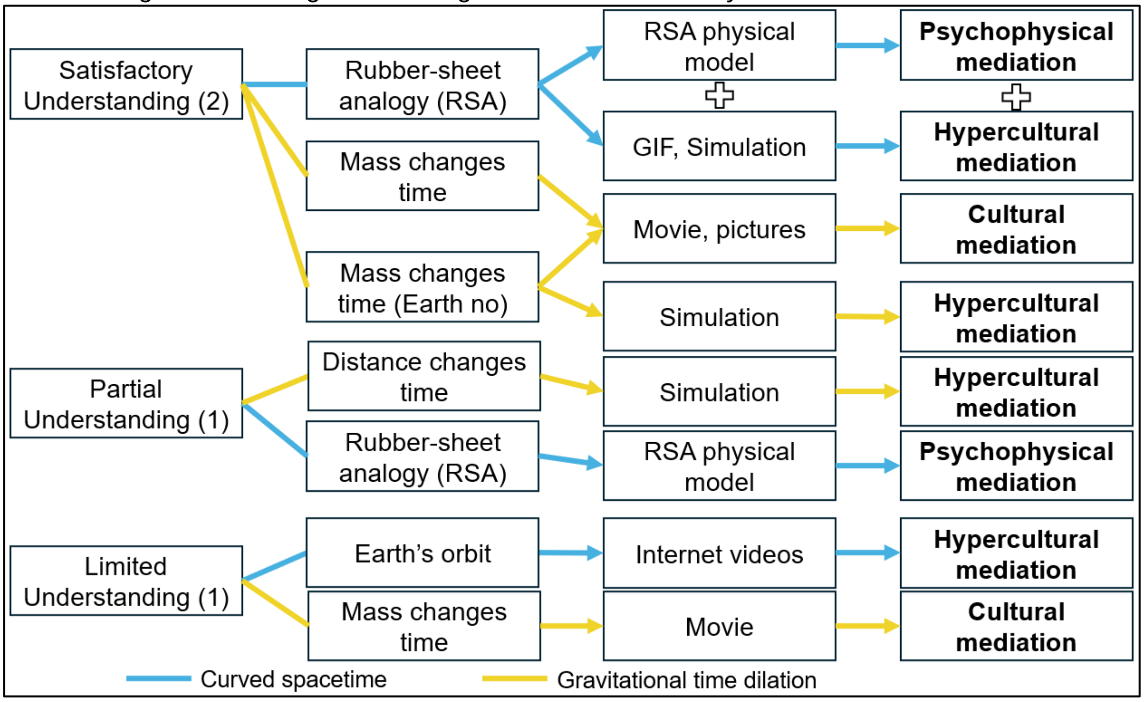
The student Mike, on the other hand, did not present indications of a mental representation or ideas about curved spacetime. Explaining Earth's movement around the Sun, Mike associated it only to the force exerted by the Sun. He demonstrated to have a pictorial mental representation of the Earth's movement, without involving curvature. As reported by Mike, probably this representation was developed through the interaction with hypercultural mediation, the internet videos mentioned by the student.

Regarding gravitational time dilation, it was possible to observe a strong influence of cultural mediation, as three of Cohort 2 students reported remembering scenes from the movie Interstellar. Interestingly, in this matter Mike, designated with limited understanding, presented a similar comprehension to Maia, satisfactory understanding. Both associated the amount of mass to time passage and reported remembering of the same scene from the movie.

Nina was also able to recognize the effects of massive objects in time passage. She also remembered a movie scene, specifically from the watches presented in this scene. However, Nina struggled to recognize Earth as a massive object capable of causing effects on time passage, even minimally. Finally, Leia presented a more limited understanding of gravitational time dilation. She recognized that mass is connected to the phenomena, but focused more on the distance, not considering the mass for some situations addressed.

Figure 10.2 presents a diagram connecting the external resources which influenced students' mental representations to the respective levels of external mediation, as reported by students in Cohort 2.

Figure 10.2: Diagram showing the mediation used by students from Cohort 2.



Source: Author (2024).

In Cohort 3, Ana explicitly linked the deformation she observed in the rubber-sheet model to her mental representation for the reason of Earth's orbit, that is the curvature caused by the Sun. The interaction with this psychophysical resource helped Ana develop a *driver* in the form of a pictorial mental representation.

However, Ana also used other levels of external mediation to construct her mental representation. She referred to the diagrams drawn during the "Embedding diagram" simulation activity (see chapter 8, section 8.1.1), indicating that this hypercultural tool also contributed to her understanding of curved spacetime. The depictive gestures made by Ana, which represented both curvature and orbital movement (see Figure 8.2, Figure 8.3 and Figure 8.4), suggest a dynamic mental representation directly linked to these psychophysical and hypercultural experiences.

Furthermore, reinforcing the use of this mental representation, Ana performed similar depictive gestures when describing her image generation with AI (see Figure

8.15, Figure 8.16 and Figure 8.17). Ana connected her image to an animated GIF used in the slides' presentation, another external hypercultural resource with which she interacted.

Similarly, although Sam had difficulty to articulate her ideas verbally, she externalized her understanding through depictive gestures that represented the rubber-sheet model (see Figure 8.10, Figure 8.11 and Figure 8.13). These gestures indicated an internalized *driver* for curved spacetime influenced by the rubber-sheet analogy. Sam demonstrated the use of the same mental representation in her image generation with AI, as observed in her depictive gestures (see Figure 8.20, Figure 8.21 and Figure 8.22). As reported by Sam, she not only used the physical rubber-sheet model, i.e., psychophysical mediation, to develop her mental representation, but also used images she searched for on the internet while researching about relativity, this is an example of hypercultural mediation.

Peter also demonstrated his possession of a mental representation of the rubber-sheet analogy, as indicated by his depictive gestures and discourse during the interview (see Figure 8.40 and Figure 8.41). However, unlike Ana and Sam, Peter did not use his mental representation in different contexts and situations that were available from the classes, such as Earth's orbit and light bending. Additionally, although the student did not directly recall during the explanation, he mentioned that the use of the physical rubber-sheet model was a memorable activity for him.

However, for students like Lara and Lisa, the rubber-sheet analogy seemed to reinforce some misconceptions they held instead of clarifying concepts. Lara, when referring to the model, attributed spatial phenomena to a force that "pulls" objects, failing to understand the idea of spacetime curvature (see Figure 8.49). This response from Lara suggests that her interaction with this psychophysical resource did not lead to the development of an appropriate *driver* for understanding the underlying principles of it.

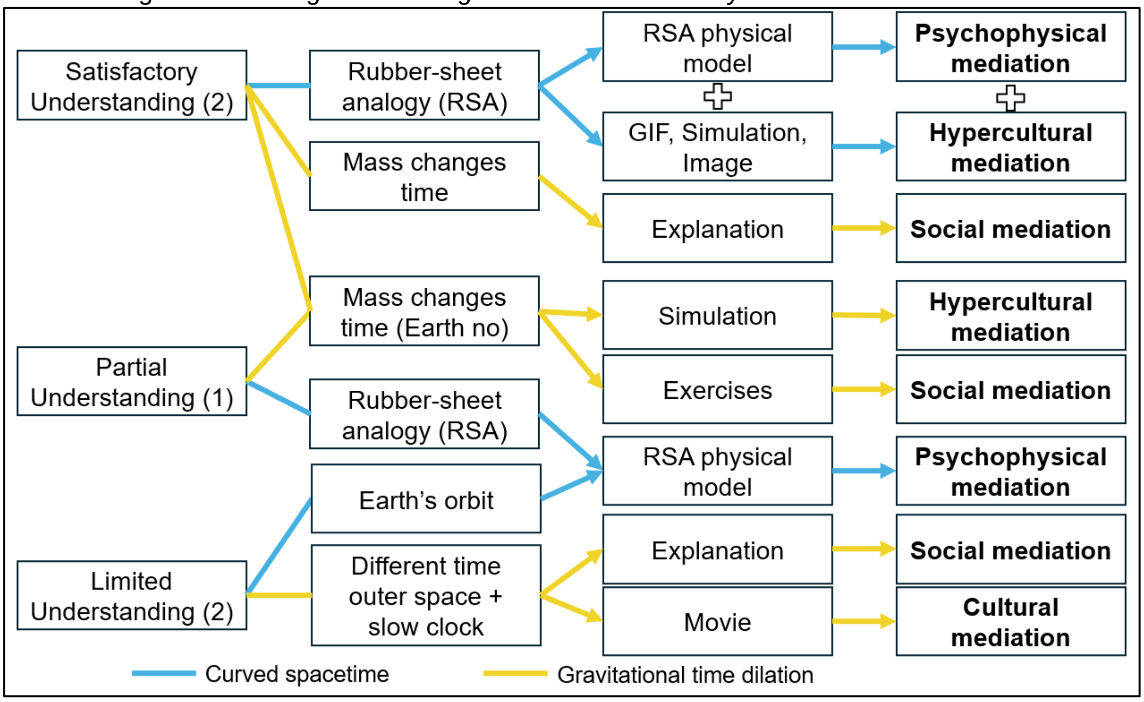
Similarly, Lisa's description of the model reflected only rote memorization instead of conceptual understanding, focusing on the observed movement of the marbles instead of the cause of that movement that is represented (see Figure 8.53). Both students demonstrated relying only on the physical rubber-sheet model (psychophysical mediation) to develop their explanations, having difficulty applying their understanding to new situations.

The prevalence of students mentioning the rubber-sheet model in their responses highlights its aspect as a memorable and tangible psychophysical mediation. Research has shown that psychophysical activities, such as practical experiments and demonstrations, have the potential to improve students' understanding of complex and counterintuitive scientific concepts. Also using the CMNT, Anjos and Serrano (2024) observed that the psychophysical mediation played an important role in students' development of the concept of light. In a conceptual change perspective, Durmuş and Bayraktar (2010) identified that experiments were successful in students' learning about matter. Finally, Urbančič and Glažar (2012) observed that students who attended to experimental science classes performed significantly better in the final test than those who did not.

This type of activity provides opportunities for learning by stimulating students' sensory experiences and facilitating the mental formulation of abstract ideas. However, although the concrete nature of the rubber-sheet model highlights a visual and tactile representation of an abstract concept, its effectiveness in fostering precise mental representations varied significantly among students.

Figure 10.3 presents a diagram summarizing the external resources and respective levels of external mediation used by students in Cohort 3 for constructing their mental representations of curved spacetime and gravitational time dilation.

Figure 10.3: Diagram showing the mediation used by students from Cohort 3.



Source: Author (2024).

Through the results, it was possible to observe potential drawbacks of excessive dependence on this single model; as students' engagement and interpretations of external mediation resources varied, influencing the development of appropriate *drivers*. Students with a good understanding of curved spacetime, such as Maia and Nina in Cohort 2, and Ana and Sam in Cohort 3, successfully integrated the physical rubber-sheet model with other mediations, such as hypercultural ones. In contrast, students with more limited understanding, such as Leia in Cohort 2, and Lara and Lisa in Cohort 3, used only this psychophysical resource in their interpretations of curved spacetime.

These contrasting results in both groups of students highlight the importance of contextualizing and integrating the rubber-sheet analogy within a broader network of external mediations. When isolated or presented without engagement in discussions, its limitations may overshadow its strengths, leading to the reinforcement of misconceptions by students. Therefore, explicit discussions about the purpose and scope of the model can make students familiar with its limitations through social mediation, for example.

Postiglione and De Angelis (2021b) highlighted that even after the activities using the rubber-sheet, some students still believed that only big masses would deform spacetime. The authors also emphasized that the misconception of gravity acting only downwards was only surpassed due to several discussions with the students about this limitation of the model. Analysing their results, Choudhary, Foppoli, Kaur, Blair *et al*. (2022) also highlighted a limitation of the rubber-sheet model to define gravity according to General Relativity, as they observed that after an intervention, many students still explaining gravity using the Newtonian conception of forces.

Furthermore, complementing the model with digital resources, an example of hypercultural mediation can provide students with a more comprehensive understanding of curved spacetime (Kersting; Steier, 2018). In the results analysed in this research, hypercultural mediations, particularly through computational simulations, offered students opportunities to actively explore the dynamics of curved spacetime. Ana, in her description of the “Embedding Diagram” simulation, demonstrated how manipulating variables and visualizing trajectories within the simulation helped her solidify her understanding of spacetime curvature. This suggests that hypercultural mediation fostered the development of a more sophisticated *driver* for curved spacetime.

In addition to psychophysical and hypercultural mediations, social and cultural mediations also played a significant role in shaping students' understanding of General Relativity. Classroom discussions provided opportunities for students to articulate their ideas, challenge each other's conceptions, and build knowledge collaboratively. For example, when discussing gravitational time dilation, Maia, Nina, Mike and Lisa referred to the scene from the *Interstellar* movie, demonstrating how a cultural artifact can serve as a structure for understanding a complex scientific concept. On the other hand, also influenced by cultural mediation, Lara's association of time dilation with "space" suggests that her understanding was tied to a specific narrative about space exploration, highlighting the importance of addressing students' everyday conceptions and linking abstract concepts to familiar contexts to them.

Social mediation, through classroom discussions and teacher explanations, also played a significant role, as evidenced when Lara stated that she remembered the teacher's explanation. This level of mediation also demonstrated impact on the mental representations of students with better understanding. Ana, for example, correctly understood that massive objects will affect time, slowing it down. She even correctly related the amount of mass to the effects caused. During the interview, Ana stated that she remembered the explanations in class.

Nina, Sam and Peter presented similar ideas, all recognizing the effects of massive objects on time passage. However, these three students demonstrated that they did not recognize the Earth as being capable of causing effects on time passage, even minimally. Like Ana, Peter relied on social mediation, as he remembered the exercises performed in class. Nina and Sam, on the other hand, recalled a computational simulation presented on the projector, therefore, using hypercultural mediation.

Through this research, it was possible to gain a comprehensive view of the intricate process by which students construct their understanding of abstract scientific concepts. The results demonstrate the potential of Cognitive Mediation Networks Theory as a reference for analysing the dynamic interaction between the external mediations and the formation of internal representations by students. The CMNT helped to explain the differences in students' ability to build and apply mental representations of concepts related to the Relativity Theory.

The greatest difference between students who were able to develop consistent mental representations and those who were not may be related to the use of only one level of external mediation for constructing their internal representations. Students with satisfactory understanding were able to develop *drivers* from interacting with different levels of mediation. These *drivers* constitute the mental representations developed to deal with phenomena not observed in everyday life.

The results suggest that effective learning of such challenging concepts is not achieved through a single mediation, but rather emerges from a cooperative combination of psychophysical, social, cultural, hypercultural, and sophotechnical experiences. This finding is in line with the CMNT's claim that efficient interaction with diverse external systems can lead to the development of internal cognitive structures (Souza, 2004; Souza; Da Silva; Da Silva; Roazzi *et al.*, 2012). Therefore, the results emphasized that a multimodal approach is necessary for scientific education; an approach that recognizes the value of diverse resources working collaboratively (De Souza; Serrano, 2020; Korhasan, 2021; Pagliarini; Almeida, 2021).

10.2 GenAI AND CONCEPTUAL UNDERSTANDING INVESTIGATION

The integration of Generative Artificial Intelligence (GenAI) tools in this research offers interesting insights into their potential for transforming physics education. GenAI represents a distinct form of mediation within the CMNT framework, called the "sophotechnic mediation". Different from other mediations, GenAI allows a dynamic creation of novel content, enabling students to externalize their mental representations and explore complex concepts in new ways.

Specifically, the use of Bing Image Creator provided a unique lens into students' internal understanding of "relativity". The image generation process, requiring students to articulate their conceptions in natural language prompts, served as a valuable form of externalization of their reasoning processes. The analysis of the students' prompts, in combination with the resulting images and students' explanations during interviews, revealed a new spectrum of analysis of students' conceptual understanding.

Students like Ana, possessing consistent pictorial mental representations of curved spacetime and good communicative skills, could use the AI tool to create visualizations aligned with accepted scientific interpretations. Their ability to translate abstract concepts into concrete visual representations demonstrates the potential of GenAI for solidifying understanding.

Interestingly, students like Sam, who possesses consistent pictorial mental representations of curved spacetime, but difficulty in express themselves verbally to use the AI tool, resorted to her gestural explanations during the interview. This observation highlights the continued relevance of physical models like the rubber-sheet, suggesting that AI tools can complement, rather than replace, existing pedagogical approaches.

Additionally, the use of AI-generated images also revealed the limitations of relying solely on visual representations for assessing understanding. Students with less developed conceptions, such as Lara and Lisa, generated images that were often abstract, ambiguous, or even contradictory to their verbal explanations. This underscores the importance of using multiple assessment methods, including interviews and gestural analysis, to gain a more complete and nuanced understanding of student thinking.

Moreover, these students' reliance on force-based conceptions when interacting with the AI suggests that careful scaffolding and guidance are crucial for ensuring that students use these powerful tools effectively. Also, this activity allowed the researcher to detect students with difficulties of interpretation, such as Sam. The same difficulty to express the ideas for image-generation also can influence students' answers in the tests.

Furthermore, the use of ChatPDF, although primarily intended for exploring confirmations of relativity theory, offered some glimpses into students' reasoning processes and alternative conceptions as they interacted with scientific literature. These results suggest the potential of AI-powered tools for formative assessment and personalized feedback in science education.

The varied students' interactions with AI tools underscore the importance of providing diverse learning opportunities and different external mediations resources when integrating GenAI into physics education. The findings of this study suggest that AI can be a powerful tool for fostering and evaluating conceptual understanding when implemented thoughtfully and in conjunction with other pedagogical strategies.

Future research should investigate the long-term impact of GenAI-mediated learning on conceptual understanding and explore the effectiveness of different AI tools across various physics topics. This includes investigating the potential of emerging GenAI technologies such as advanced AI-powered search engines capable of synthesizing information from diverse sources, tools enabling nuanced interaction with various file types for data extraction and analysis, and large reasoning models like the o1 family from OpenAI (Openai, 2024).

These new models demonstrate improved planning and reasoning capabilities and have the potential to change the landscape of physics teaching even further than the previous models. Developing efficient methods for assessing students' learning in these novel, AI-augmented environments will be essential for maximizing their educational potential and promoting scientific literacy in the digital age. Further investigations should also focus on the potential of these advanced AI tools to address alternative conceptions and misconceptions effectively.

10.3 LIMITATIONS AND SUGGESTIONS FOR FUTURE RESEARCH

Although this research provides relevant information and brings important perspectives on the mental representations constructed by students and their conceptual understanding of Relativity Theory, some limitations must be acknowledged.

Firstly, comparing to a broader context, the sample used may be considered small, with a total of 32 students: 14 students in Cohort 1, 8 students in Cohort 2, and 10 students in Cohort 3. Additionally, the three data collection experiments were conducted at the same school and at the same level of education. This limited sample and demographic range may not be fully representative of the broader student population, potentially affecting the generalizability of the results.

It is also necessary to highlight the short period of time that students had contact learning Relativity Theory. Students in Cohort 1 participated in 6 classes of 1h 40 min, while students in Cohort 2 and 3 participated in 11 classes of 1 h 30 min each. Different patterns of development and retention of mental representations may be identified through a long-term study.

Although Cohort 1 was conducted during regular physics classes, Cohort 2 and 3 were developed in an extracurricular environment. The offer of the extracurricular course may have attracted students with a pre-existing interest in physics, potentially affecting the results compared to a standard classroom environment.

Although the sample and demographic range are limited, since each data collection was conducted in a different year (2019, 2022, and 2023), this may have provided a more comprehensive range of cases as well as the characteristics common over these years. Additionally, Cohort 1 was conducted before the COVID-19 pandemic, while Cohort 2 and 3 were conducted afterwards, bringing students with different prior learning experiences, although from the same school.

Research have found different negative effects of COVID-19 pandemic on students learning, mainly in low socioeconomic backgrounds (Tan, 2021). Considering the students who participated in this research, comparing the students from 2022 and 2023 to the ones from 2019, they showed no long-term learning problems as they effectively engaged in the activities and accepted the cognitive challenges of the course. Evidently, this may be potentialized by pre-existing interests in physics, as mentioned earlier, because it was an extracurricular course.

Furthermore, considering the limitations mentioned, some precautions were taken to ensure the validity of the results. Firstly, to provide a broader context for the presented results, they were consistently compared with existing studies on teaching and learning of Special and General Relativity.

Multiple data collection methods were used, including pre-tests and post-tests, interviews, gestural analysis, discourse analysis, and materials produced by students, such as diagrams and AI images. This allowed for a comprehensive understanding of the data, performing cross-verification of the results. Additionally, to provide a nuanced analysis, detailed descriptions of the students' responses were presented, including their verbal, gestural, and written expressions.

The use of the *Report Aloud* protocol for conducting interviews, which allowed students to elaborate and explain their thoughts and ideas, enabled verification of their written responses. Regarding the gestural analysis, the interview allowed a confirmation from the students about the gesture interpretation. Additionally, the findings were discussed among the research group members, to provide reliability in the analysis. Furthermore, the researcher who conducted the experiments and interviews was also the physics teacher of the students. This established a relationship with the participants, potentially allowing for more open and honest responses.

Therefore, considering the presented results, this research serves as a starting point for a more in-depth exploration of the application of CMNT in scientific education. Future research may investigate the long-term retention and application of mental representations formed through various levels of external mediations.

Additionally, investigating how specific combinations of external resources foster robust mental models may provide valuable insights for curriculum planning and pedagogical practices. For example, considering the curved spacetime concept, the use of psychophysical and hypercultural mediations combined seemed to be efficient, as evidenced by Ana's and Sam's results. Considering time dilation, for both Special and General Relativity, the social mediation played an important role, as all students who seemed some understanding of it reported.

Finally, the scope of this research can be expanded to other abstract or counterintuitive concepts in scientific disciplines. Moreover, future research in different schools and with more participants might confirm, or not, the results presented here. Such research findings would expand the understanding and potential generalizability of CMNT, as well as promote the inclusion of modern and contemporary physics topics in school education.

10.4 IMPLICATIONS FOR PHYSICS TEACHING

This doctoral research contributes to the field of investigation on students' mental representations of complex scientific topics (Ubben; Hartmann; Pusch, 2022). Through the analysis of the results, we can observe the outstanding importance of using multiple external mediation levels to assist students' conceptual understanding.

Analysing the development and use of students' mental representations for understanding Einstein's Relativity Theory opens a window into students' cognitive processes in learning, extending existing studies on the development of mental models in scientific education (Batlolona; Diantoro; Wartono; Leasa, 2020; Fratiwi; Samsudin; Ramalis; Saregar *et al.*, 2020; Özcan, 2013; Saglam-Arslan; Karal; Akbulut, 2020; Ubben; Heusler, 2021).

The results presented emphasize the need for a multimodal approach to science education, recognizing the value of diverse resources working collaboratively. This perspective may enable students to navigate between different levels of abstraction and express their reasoning processes effectively. Therefore, educators should encourage students to articulate their mental representations through multiple modes (verbal, gestural, drawings) to evaluate their conceptual understanding of these concepts. According to Prain and Tytler (2022) the students' reasoning across these different modes can be used to observe their science learning. Moreover, Kersting, Danielsson, Mortimer, Olander *et al*. (2024) highlighted that the idea of "science literacy" evolved to a broader view using multimodal means of communication.

Moreover, the results obtained also bring significant implications for teaching abstract concepts in physics classes and beyond. By using the CMNT as a guiding theory, educators can achieve a more refined and effective understanding of how students construct meaning from complex scientific ideas. Therefore, lesson planning should encourage the use of external tools, exploring all levels of mediation, that allow students to gain cognitive benefits through their usage (Souza; Da Silva; Da Silva; Roazzi *et al.*, 2012; Souza; Serrano; Roazzi, 2024).

This perspective encourages a paradigm shift in teaching approach, aiming to foster cognitive processes. This approach can fill the gap between the school curricula and the modern world (Kersting; Blair, 2021), mentioned in the rationale of this thesis, promoting scientific literacy and curiosity. The use of the strategies presented here might enable to align school instruction with the best scientific understanding of society, what can improve students' attitudes towards science (Foppoli; Choudhary; Blair; Kaur *et al.*, 2019; Vakarou; Stylos; Kotsis, 2024).

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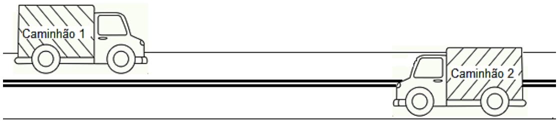
APPENDIX

APPENDIX A – Final version of the pre/post-test

Name: ______________________________________ Date: ___/___/______

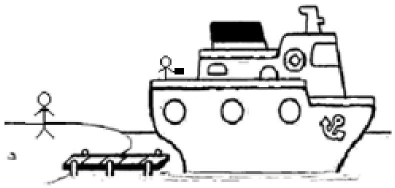
Questions 1 and 2: Two trucks are on the same road, in opposite directions, as the figure. The truck 2 is stopped due to an engine problem. The distance between them, measured by the truck 2, is 100 m.



1) If the truck 1 is moving at 80 km/h, the distance measured by it will be, approximately:
   a) Also 100 m.
   b) Little bigger than 100 m.
   c) Much bigger than 100 m.
   d) Little smaller than 100 m.
   e) Much smaller than 100 m.

2) Now, considering that truck 1 is moving with a huge speed (0,7c), the distance measured by it will be, approximately:
   a) Also 100 m.
   b) Little bigger than 100 m.
   c) Much bigger than 100 m.
   d) Little smaller than 100 m.
   e) Much smaller than 100 m.

Questions 3 and 4: A person is traveling by ship and performs an experiment while the ship passes the coast. A person on the coast observes and notes that the experiment takes 10 s to be done.



3) If the ship travels at a speed of 30 knots (56 km/h), the time interval to the little ball returns to person's on the ship hand will be, approximately:
   a) Also 10 s.
   b) Little bigger than 10 s.
   c) Much bigger than 10 s.
   d) Little smaller than 10 s.
   e) Much smaller than 10 s.

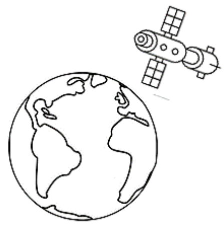
4) Now, if the travels at a huge speed (0,6c), the time interval to the little ball returns to person's on the ship hand will be, approximately:
   a) Also 10 s.
   b) Little bigger than 10 s.
   c) Much bigger than 10 s.
   d) Little smaller than 10 s.
   e) Much smaller than 10 s.

Questions 5 and 6: Consider two friends that have the same age. One of them goes to a travel by cruise, while the other one stills on the city. On the return from the trip, it passed one year for the friend who stayed at the city.

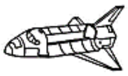
5) If the cruise travels with a speed of 30 knots (56 km/h), the time interval for the friend who travelled will be:
   a) Also 1 year.
   b) Little bigger than 1 year.
   c) Much bigger than 1 year.
   d) Little smaller than 1 year.
   e) Much smaller than 1 year.

6) Now, if the cruise travels with a huge speed (0,5c), the time interval for the friend who travelled will be:
   a) Also 1 year.
   b) Little bigger than 1 year.
   c) Much bigger than 1 year.
   d) Littler smaller than 1 year.
   e) Much smaller than 1 year.

7) Two astronauts are in the international space station. Consider that one of them goes down to Earth, do an activity and finished it in 8 h, according to his watch. What was the time interval for the astronaut who stayed in the space station?



a) Also 8 h.
b) Little more than 8 h.
c) Much more than 8 h.
d) Little less than 8 h.
e) Much less than 8 h.



8) Consider now that the astronauts are on a spaceship close to a supermassive black hole. One of them goes in a mission and gets closer to the black hole, finishing the mission in 3 h, according to his watch. What was the time interval for the astronaut who stayed on the spaceship?

a) Also 3 h.
b) Little more than 3 h.
c) Much more than 3 h.
d) Little less than 3 h.
e) Much less than 3 h.

9) Two points close to the Earth have between them a distance of 1 km. What would be the distance between these points if they were close to a black hole in an altitude of 500 km?

a) Also 1 km.
b) Little more than 1 km.
c) Much more than 1 km.
d) Little less than 1 km.
e) Much less than 1 km.

10) Two points close to the Earth have between them a distance of 1 km. What would be the distance between these points if they were close to a black hole in an altitude of 20 km?

a) Also 1 km.
b) Little more than 1 km.
c) Much more than 1 km.
d) Little less than 1 km.
e) Much less than 1 km.

11) Explain, as you were telling to a classmate, why the Earth moves in an orbit around the Sun. To do that, you can use text, drawing, graphics, diagrams etc.

APPENDIX B – Simulation guide: Galileo's Relativity

Names: ____________________________________ Date: ___/___/______

In this activity we are going to use two computer simulations with the software *Modellus*, where we are going to observe the movement of some objects from different perspectives.

Ball on the Train Simulation

A boy inside a train throws a little ball ahead with a velocity of 5 km/h related to himself. The train is moving with a velocity of 50 km/h according to the ground.

1st Situation

Explain how the ball's movement (trajectory, velocity etc.) will be observed by a person who is outside the train, in the station. To do that, you can use text, drawing, diagrams and/or graphics.

Describe what you observed in the simulation. To do that, you can use text, drawing, diagrams and/or graphics.

Explain the differences and similarities between what you imagined that would happen and what you observed in the simulation. To do that, you can use text, drawing, diagrams and/or graphics.

2nd Situation

Now, explain how the ball's movement (trajectory, velocity etc.) will be observed by the boy inside the train. To do that, you can use text, drawing, diagrams and/or graphics.

Describe what you observed in the simulation. To do that, you can use text, drawing, diagrams and/or graphics.

Explain the differences and similarities between what you imagined that would happen and what you observed in the simulation. To do that, you can use text, drawing, diagrams and/or graphics.

Simulation Cars and Plane

In this simulation there are two cars, a plane and people on the sidewalk. The plane is moving with a velocity of 400 km/h to the right side, the green car is moving with a velocity of 50 km/h also to the right side, the black car is moving with a velocity of 80 km/h to the left side and the people is at rest in the sidewalk, all according to the ground.

What is the black car's velocity according to the plane?

<u>1st Situation</u>

Explain what will happen if the plane's velocity is reduced to 100 km/h (what will be the velocity of the cars according to the plane etc.). To do that, you can use text, drawing, diagrams and/or graphics.

Describe what you observed in the simulation. To do that, you can use text, drawing, diagrams and/or graphics.

Explain the differences and similarities between what you imagined that would happen and what you observed in the simulation. To do that, you can use text, drawing, diagrams and/or graphics.

2nd Situation

Explain how you imagine that this same situation would be observed by someone inside the plane (what will be the velocity of the cars according to the plane etc.). To do that, you can use text, drawing, diagrams and/or graphics.

Describe what you observed in the simulation. To do that, you can use text, drawing, diagrams and/or graphics.

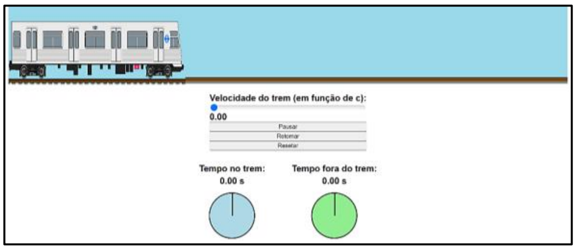
Explain the differences and similarities between what you imagined that would happen and what you observed in the simulation. To do that, you can use text, drawing, diagrams and/or graphics.

APPENDIX C – Simulation guide: Special Relativity

Names: _______________________________________ Date: ___/___/______

In this activity we ae going to use two simulations to observe the *Time Dilation* and *Space Contraction* phenomena for *Special Relativity* as we saw on classes.

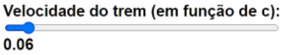
Time Dilation

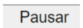


First, let's observe the Time Dilation simulation. To that, open the folder *Time* and, inside the folder, the file "*index*" with double-click. On simulation there is a train moving in relation to the station. It is possible to observe the time passing inside the train (Δt_0) and on a reference frame at rest related to the ground (Δt) (station). Both time intervals are observed from the station reference frame.

<u>1st Situation</u>

Explain how the time passage inside and outside the train, for a reference frame on the station, will be when it is moving at 0,06c related to the ground. Will there have differences between the two measurements? Explain. To that, you can use text, drawing, diagrams and/or graphics

Now let's observe on the simulation. To that, the train's velocity must be adjusted to 0,06c in the slide controller which must be in the following way: 

. While adjusting the velocity, the train will start moving. Observe the simulation until the train disappear of the screen. In this moment, pause the simulation by clicking on "*Pausar*" 

.

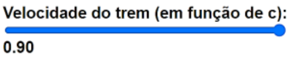
Describe how you observed the time passage in the simulation. To that, you can use text, drawing, diagrams and/or graphics.

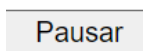
Explain the differences and similarities between what you imagined and what you observed in the simulation. To that, you can use text, drawing, diagrams and/or graphics.

<u>2nd Situation</u>

Now, explain how the time passage inside and outside the the train, for a reference frame in the station, will be when it is moving at 0,98c related to the ground. Will there have differences between the two measurements? To that, you can use text, drawing, diagrams and/or graphics.

Let's observe on the simulation. First, we must reset the simulation, for that, click on *"Resetar"* 

. Now, adjust the train's velocity to 0,9c in the slide controller which must be in the following way: 

. While adjusting the velocity, the train will start moving. Observe the simulation until the train disappear of the screen. In this moment, pause the simulation by clicking on "*Pausar*" 

.

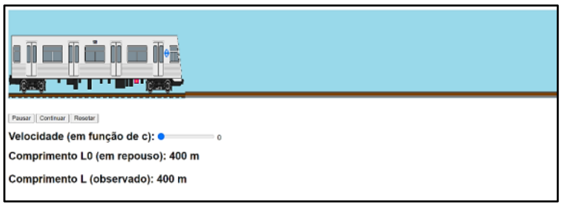
Describe how you observed the time passage in the simulation. To that, you can use text, drawing, diagrams and/or graphics.

Explain the differences and similarities between what you imagined and what you observed in the simulation. To that, you can use text, drawing, diagrams and/or graphics.

Close the simulation clicking on *Close* on the top right corner of the screen.

Answer: Observing the simulation, for which reference frame the time passed slower? In which of the situations was the time difference bigger? Why do you imagine that this happened?

Space Contraction



Now, let's serve the Space Contraction simulation. For that, open the folder *Space*, inside the folder, open the file "*index*" with double-click. On this simulation there is also a train moving related to the ground. It is possible to observe the measured length outside the train, someone at rest related to the ground (L) (station), and the measured length inside the train, for someone at rest related to the train (L_0).

1st Situation

Explain how the train's length will be measured for the reference frames inside and outside it when it is moving at 0,06c related to the ground. Will there have differences? Explain. To that, you can use text, drawing, diagrams and/or graphics.

Now, let's observe on the simulation. Before we start, click on "*Resetar*" Resetar. Now, verify if the train's velocity is adjusted to 0,06c in the controller which must be in the following way: **Velocidade (em função de c):** 0.06. While adjusting the velocity, the train will start moving. Observe the simulation until the train disappear of the screen. In this moment, pause the simulation by clicking on "*Pausar*" Pausar. Describe how you observed the time passage in the simulation. To that, you can use text, drawing, diagrams and/or graphics.

Explain the differences and similarities between what you imagined and what you observed in the simulation. To that, you can use text, drawing, diagrams and/or graphics.

2nd Situation

Now, explain how the train's length will be measured for the reference frames inside and outside it when it is moving at 0,98c related to the ground. Will there have differences? Explain. To that, you can use text, drawing, diagrams and/or graphics.

Let's observe on the simulation. First, we must reset the simulation, for that, click on *"Resetar"* Resetar. Now, adjust the train's velocity to 0,9c in the slide controller which must be in the following way: **Velocidade (em função de c):** 0.90. While adjusting the velocity, the train will start moving. Observe the simulation until the train disappear of the screen. In this moment, pause the simulation by clicking on "*Pausar*" Pausar Describe how you observed the time passage in the simulation. To that, you can use text, drawing, diagrams and/or graphics s.

Explain the differences and similarities between what you imagined and what you observed in the simulation. To that, you can use text, drawing, diagrams and/or graphics.

As before, close the simulation clicking on *Close* on the top right corner of the screen.

Answer: For which reference frame was the length smaller? On each of the situations was the difference bigger? Why do you imagine that this happened?

APPENDIX D – Simulation guide: Gravitational Time Dilation

Names: ____________________________________ Date: ___/___/______

Let's use the simulation called *GR Time Dilation*, that deals with the Gravitational Time Dilation.

1st Situation

How the time passage will be on each of the points? Will there have differences? What? Explain. To that, you can use text, drawing, diagrams and/or graphics.

Now, let's observe on the simulation. Describe how the time passage for each point was. To that, you can use text, drawing, diagrams and/or graphics.

Explain the differences and similarities between what you imagined and what you observed in the simulation. To that, you can use text, drawing, diagrams and/or graphics.

2nd Situation

Now, how will the time passage be if the green point were put away from the massive object? Will there have differences? What? Explain. To that, you can use text, drawing, diagrams and/or graphics.

Let's observe on the simulation. Describe how the time passage for each point was. To that, you can use text, drawing, diagrams and/or graphics.

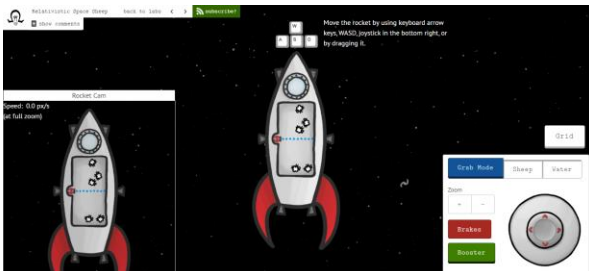
Explain the differences and similarities between what you imagined and what you observed in the simulation. To that, you can use text, drawing, diagrams and/or graphics.

APPENDIX E – Simulation guide: Relativistic Space Sheep

Names: _______________________________________ Date: ___/___/______

In this activity we are going to use a simulation called *Relativistic Space Sheep*, which approaches the Equivalence's Principle, as we saw on classes, that affirms that the effects of the experiments made on a gravitational field would be locally the same as the ones perceived on an accelerated reference frame.

To access the simulation, open the browser and use the hyperlink: http://labs.minutelabs.io/Relativistic-Space-Sheep/. The initial screen simulation will be charged as the image bellow.



In the simulation, there is a rocket at rest. Inside it there is five sheep and a water bomb that throws water drops that cross the rocket. The sheep don't know they are inside a rocket.

- What is the behaviour of the sheep and water drops when the rocket is at rest?

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- On the sheep's perspective, is there a gravitational field in the rocket? Explain.

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1st Situation

Explain how the sheep and water drops behaviour will be if the rocket started accelerating upwards. To that, you can use text, drawing, diagrams and/or graphic.

Now, let's observe on the simulation. First, verify if the rocket is at rest, if now, use the brakes clicking on *Brakes* 

, in the bottom right corner of the screen. To move the rocket upwards the bottom engines must be activated. For this, use the key "w" or the arrow upwards of the keyboard. For the rocket stay accelerating, the key must still press.

Describe the behaviour of the sheep and water drops that you observed on the simulation. To that, you can use text, drawing, diagrams and/or graphic.

- On this situation, on the sheep's perspective, is there a gravitational field in the rocket? Explain.

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Explain the differences and similarities between what you imagined and what you observed in the simulation. To that, you can use text, drawing, diagrams and/or graphics.

<u>2nd Situation</u>

Now, explain how the sheep and water drops behaviour will be if the rocket started accelerating downwards. To that, you can use text, drawing, diagrams and/or graphics.

Let's observe on the simulation. First, verify if the rocket is at rest, if now, use the brakes clicking on *Brakes* 

, in the bottom right corner of the screen. To move the rocket downwards the top engines must be activated. For this, use the key "s" or the arrow downwards of the keyboard. For the rocket stay accelerating, the key must still press.

Describe the behaviour of the sheep and water drops that you observed on the simulation. To that, you can use text, drawing, diagrams and/or graphics.

- On this situation, on the sheep's perspective, is there a gravitational field in the rocket? Explain.

Explain the differences and similarities between what you imagined and what you observed in the simulation. To that, you can use text, drawing, diagrams and/or graphics.

<u>3rd Situation</u>

Explain how the sheep and water drops behaviour will be if the rocket started moving upwards with a constant velocity. To that, you can use text, drawing, diagrams and/or graphics.

Let's observe on the simulation. First, verify if the rocket is at rest, if now, use the brakes clicking on *Brakes* 

, in the bottom right corner of the screen. To move the rocket upwards the bottom engines must be activated. For this, use the key "w" or the arrow upwards of the keyboard. As we want that the rocket moves at a constant velocity, i.e., without acceleration, click on the key Only once and observe without pressing it.

Describe the behaviour of the sheep and water drops that you observed on the simulation. To that, you can use text, drawing, diagrams and/or graphics.

- On this situation, on the sheep's perspective, is there a gravitational field in the rocket? Explain.

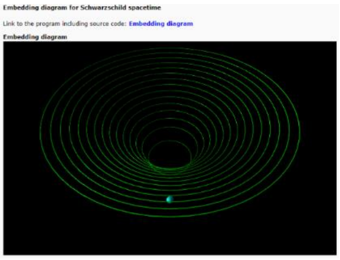
Explain the differences and similarities between what you imagined and what you observed in the simulation. To that, you can use text, drawing, diagrams and/or graphics.

Finishing the activity, close the browser's window.

APPENDIX F – Simulation guide: Embedding Diagram

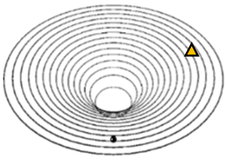
Names: ____ Date: ___/___/______

For this activity we are going to use a simulation called *Embedding diagram for Schwarzschild spacetime*, which delas with the curvature caused by a massive object, as we saw on classes, that is a prevision of Einstein's General Relativity Theory. To access the simulation, open the browser a use the hyperlink: http://kdf.mff.cuni.cz/~ryston/embed_en.php. The initial simulation's screen will be loaded as the image.

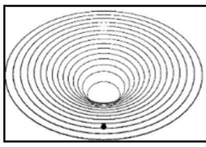


On simulation, the green lines represent the geodesics and show the curvature caused by a massive object, which is Hidden. First, let's make this object visible, to do that, bellow the simulation's screen there are many controllers. Click on the *Show central mass* [Show central mass], the last one on the first line. A yellow sphere will appear in the middle of the geodesics, it is the massive object causing the observed curvature. There is also a small blue sphere, that represents the less massive object that can move on space.

<u>1st Situation</u>

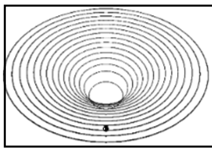


Explain how the blue sphere movement will be (the trajectory, velocity etc.) if it was thrown in the direction of the triangle on the image. For that, you can use the image bellow to show the trajectory or use text, drawing, diagrams and/or graphics.



Let's observe on the simulation. To move the blue sphere, you must click on it and, with the key pressed, slide the mouse towards the indicated point. In doing so, a red arrow will appear, it represents the initial velocity vector of the blue sphere. When you stop pressing the key, the red arrow will disappear, and the sphere will move.

Describe the blue sphere movement that you observed on simulation. For that, you can use the image bellow to show the trajectory or use text, drawing, diagrams and/or graphics.

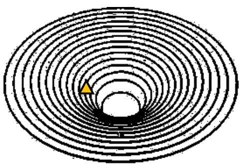


Explain the differences and similarities between what you imagined and what you observed on simulation. For that, you can use text, drawing, diagrams and/or graphics.

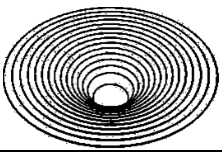
Let's change the blue sphere initial position. First, let's clean the later trajectory. For that, click on *Clear trajectories* [Clear trajectories], the last button on the second line bellow the simulation. To change the blue sphere initial position, there is a slide controller bellow the buttons called *Initial radius*. Drag the slide controller until it be on the position showed on the figure:

Note that the blue sphere got closer of the yellow one, i.e., we decreased the initial radius.

<u>2nd Situation</u>

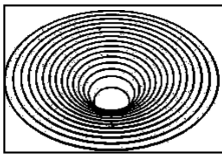


Explain how the blue sphere movement will be (the trajectory, velocity etc.) if it was thrown in the direction of the triangle on this image. For that, you can use the image bellow to show the trajectory or use text, drawing, diagrams and/or graphics.



Let's observe on the simulation. Again, to move the blue sphere, you must click on it and, with the key pressed, slide the mouse towards the indicated point.

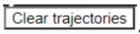
Describe the blue sphere movement that you observed on simulation. For that, you can use the image bellow to show the trajectory or use text, drawing, diagrams and/or graphics.



Explain the differences and similarities between what you imagined and what you observed on simulation. For that, you can use text, drawing, diagrams and/or graphics.

What were the differences of the blue sphere movements on the 1st and 2nd situation? What is the reason for this? ____

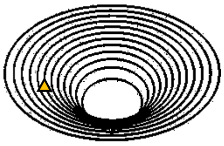
____

Let's change the yellow sphere's mass. First, let's clean the later trajectory. For that, click on *Clear trajectories* 

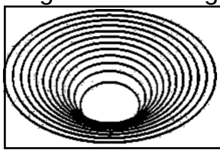
. To change the yellow sphere's mass, there is a slide controller bellow the buttons called *Central mass*. Drag the slide controller until it be on the position showed on the figure:

Note that the yellow sphere's mass increased. Let's also return the blue sphere to the initial position of the first situation. Vamos também retornar a esfera azul para a posição inicial da primeira situação. To that, drag the slide controller *Initial radius* until it be on the position showed on the figure: 

<u>3rd Situation</u>

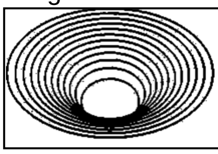


Now, in this situation, explain how the blue sphere movement will be (the trajectory, velocity etc.) if it was thrown in the direction of the triangle on this image. For that, you can use the image bellow to show the trajectory or use text, drawing, diagrams and/or graphics.



Let's observe on the simulation. Again, to move the blue sphere, you must click on it and, with the key pressed, slide the mouse towards the indicated point.

Describe the blue sphere movement that you observed on simulation. For that, you can use the image bellow to show the trajectory or use text, drawing, diagrams and/or graphics.



Explain the differences and similarities between what you imagined and what you observed on simulation. For that, you can use text, drawing, diagrams and/or graphics.

What were the differences of the blue sphere movements on the 3rd and 1st situation? What is the reason for this? ____

____

Finishing the activity, close the browser's window.

APPENDIX G – Simulation Guide: Curved Spacetime

Names: ______________________________________ Date: ___/___/______

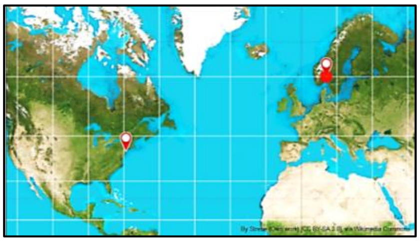
In this activity we are going to use the virtual environment called *General Relativity,* we are going to use on the module 3: *Curved Spacetime.* Let's make some activities of this module. Open the browser and use the hyperlink: https://www.viten.no/filarkiv/general-relativity/#/. The initial screen will be loaded. Scrolling the page, it is possible to see the modules in blue blocks. Open the module 3 by clicking on *Open module*.

Scrolling the page again, is possible to see the proposed activities. The first part has some text and interactive images. Read the texts carefully. On the topic *Geometry in curved spaces* there is an interactive activity. To start if, click on the arrow on the right side.

On this activity, there will be some instructions above the images that appear. On the first part, you must draw what would be the shorter path to a plane travel from Oslo to New York, both marked on the map.

Draw on the map bellow the path between the two cities that you consider the shorter one.

1)



Try to do the same path on computer screen, for that, click on the red dot and with the key pressed, drag it. After that, click on the button *show answer* mostre a resposta, on the bottom right corner, bellow the map.

Did you note differences between the path you draw and the answer? If yes, what is the reason for it?

____

____

Click on the right arrow to continue activity. Now, there is a different kind of map. Again, draw on the map the path between the two cities that you consider the shorter one.

2)

Try to do the same path on computer screen, for that, click on the red dot and with the key pressed, drag it. After that, click on the button *show answer* mostre a resposta, on the bottom right corner, bellow the map.

Did you note differences between the path you draw and the answer? If yes, what is the reason for it?

____

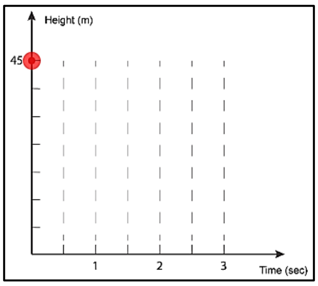
____

Scroll the screen down to the activity *Curved time*. Read the small introduction text, bellow, there is an image of the *Gingin* tower*,* open the marks on the image, from bottom to top, read the texts and, after that, click on the right arrow.

Consider that Einstein stay on the tower top. On the right there is a diagram of the height in function of time, draw the line on diagram (3) that represents Einstein's movement.

Try to do the same line on computer screen, for that, click on the red dot and with the key pressed, drag it. After that, click on the button *show answer* mostre a resposta, on the bottom right corner, bellow the diagram.

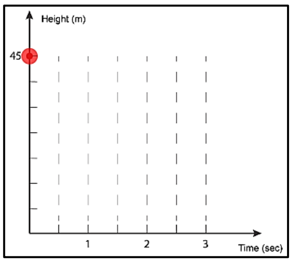
3)



Did you note differences between the path you draw and the answer? If yes, what is the reason for it?

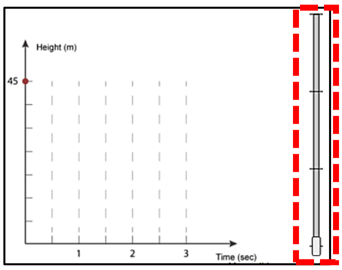
Click on the right arrow to continue. On next situation, Einstein jumps from the tower. Again, <u>draw</u> the line on diagram (4) that represents Einstein's movement.

Try to do the same line on computer screen, for that, click on the red dot and with the key pressed, drag it. After that, click on the button *show answer* mostre a resposta, on the bottom right corner, bellow the diagram.



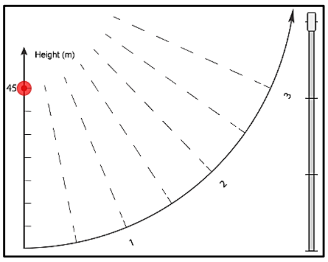
4)

Did you note differences between the path you draw and the answer? If yes, what is the reason for it?



Click on the right arrow to continue. Now the diagram can be changed. To do this, you must drag the *warp time*, a slide vertical controller on the right of the diagram (indicated by the red dashed line). Drag it until the top (like to the image below). Again, consider that Einstein <u>stay on the tower top</u>. <u>Draw</u> a line on the diagram (5) that represents Einstein's movement.

Try to do the same line on computer screen, for that, click on the red dot and with the key pressed, drag it. After that, click on the button *show answer* mostre a resposta, on the bottom right corner, bellow the diagram.



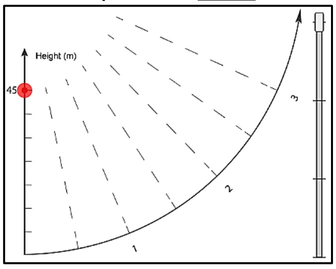
5)

Did you note differences between the path you draw and the answer? If yes, what is the reason for it?

Click on the right arrow to continue. Again, drag the *warp time* until the top. Now, <u>draw</u> on the diagram (6) a line representing when Einstein jumps from the tower.

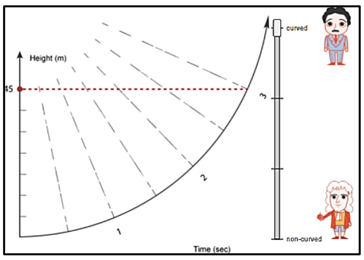
Try to do the same line on computer screen, for that, click on the red dot and with the key pressed, drag it. After that, click on the button *show answer* mostre a resposta, on the bottom right corner, bellow the diagram.

Did you note differences between the path you draw and the answer? If yes, what is the reason for it?



6)

Click on the right arrow to continue. Now, it is possible to visualize the diagram with the line already drawn and drag the *warp time* seeing the same situation (Einstein on the top of tower and jumping from it) on Einstein's (top) and Newton's (bottom) perspectives.



Click on the right arrow to continue. Read the text *Gravity is not a force* and observe the animated gif representing the gravitational time dilation. Scroll the screen and read the *Summary – warped time*.

Ps: We are not going to do the last part of the module.

Finishing the activity, close the browser window.

APPENDIX H – Use guide: Bing AI

Name: ____________________________________ Date: ___/___/______

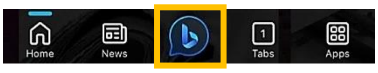
In this activity you are going to create a description from an image and generate it with the Bing AI. The image you are going to generate is of what you imagine when think about the word "***relativity***".

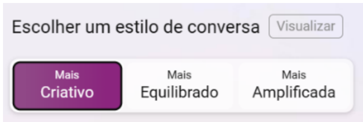
- Describe bellow, through text, what you imagine when you think on the word ***"relativity"***. Don't be concern about your answer, just describe what you spontaneously imagine.

Now, let's use the Bing AI to create the image.

- Open the Bing app on your smartphone and log0in on your Microsoft account.

To write the commands (prompts), we must use the chat space. You can access it clicking on the middle of the bottom part of the screen, as the image.





- On the option "Chat style", choose the "creative style", according to image.
- On chat, write the following sentence: **"Generate an image…"** and put the description of what you imagined. After, send the message clicking on .

As much detailed the description better is the image!

Four images will be generated. Look them and choose the one that is closer to what you imagined. You can click on the images to open them amplified. Send the image that you selected to the teacher with your full name (*maira.souza@rede.ulbra.br*).

- Does the image look like what you imagined? On what it is similar and on what is not?

- What do you think that could be modified on your description to the image be closer to what you imagined?

- Ask Bing to make the changes that you think are necessary, it is just send it on the chat.

Send this new image to the teacher with your full name (*maira.souza@rede.ulbra.br*).

APPENDIX I – Permission letter

ESTADO DO RIO GRANDE DO SUL
SECRETARIA ESTADUAL DA EDUCAÇÃO
2ª COORDENADORIA REGIONAL DE EDUCAÇÃO - SÃO LEOPOLDO
ESCOLA ESTADUAL TÉCNICA SÃO JOÃO BATISTA

Rua João Pessoa, 1468 - CEP 95780-000 - Fone/Fax: (0**51) 3632-1709 / 3632-5001
e-mail: eesjb@terra.com.br - MONTENEGRO - Rio Grande do Sul

CARTA DE ANUÊNCIA

Ao Comitê de Ética em Pesquisa em Seres Humanos da Universidade Luterana do Brasil/RS

Prezados Senhores,

Declaro que tenho conhecimento e autorizo a realização do projeto de pesquisa intitulado "A TEORIA DA RELATIVIDADE GERAL NO ENSINO MÉDIO: UM ESTUDO SOBRE AS REPRESENTAÇÕES MENTAIS DOS ESTUDANTES", proposto pela pesquisadora MAIRA GIOVANA DE SOUZA. O referido projeto será realizado na Escola Estadual Técnica São João Batista onde será realizado o estudo, e só poderá ocorrer a partir da apresentação do Parecer de Aprovação do Colegiado do Comitê de Ética em Pesquisa em Seres Humanos da Universidade Luterana do Brasil/RS.

Montenegro, em 24/09/2021.

Responsável pela Instituição

Juliana Cabreira Bender
DF 1847905/01 - DO 27-12-2018
DIRETORA

APPENDIX J – Data Usage Commitment Term

ULBRA
UNIVERSIDADE LUTERANA DO BRASIL
Reconhecida pela Portaria Ministerial nº 906 de 17/08/2016 – D.O.U. de 18/08/2016
ASSOCIAÇÃO EDUCACIONAL LUTERANA DO BRASIL
PRÓ-REITORIA ACADÊMICA
DIRETORIA DE PÓS-GRADUAÇÃO E PESQUISA
COMITÊ DE ÉTICA EM PESQUISA EM SERES HUMANOS

TERMO DE COMPROMISSO PARA A UTILIZAÇÃO DE DADOS

Título do projeto: A TEORIA DA RELATIVIDADE GERAL NO ENSINO MÉDIO: UM ESTUDO SOBRE AS REPRESENTAÇÕES MENTAIS DOS ESTUDANTES

Os autores do projeto de pesquisa se comprometem a manter o sigilo dos dados coletados referentes aos participantes atendidos na **Escola Estadual Técnica São João Batista**.

Concordam, igualmente, que estas informações serão utilizadas única e exclusivamente com finalidade científica, preservando-se integralmente o anonimato dos participantes.

CANOAS, 07 de outubro de 2021

Autores do Projeto	
Nome	**Assinatura**
Maira Giovana de Souza	Maira Giovana de Souza
Agostinho Serrano de Andrade Neto	Agostinho Serrano de A. Neto

Rua Farroupilha, 8001 – Prédio 14 – Sala 224 - Bairro São José - Canoas/RS - CEP 92.425-900
Fone: (51)3477-9217 - E-mail: comitedeetica@ulbra.br - Home Page: www.ulbra.br/pesquisa

PPGECIM
PROGRAMA DE PÓS-GRADUAÇÃO EM ENSINO DE CIÊNCIAS E MATEMÁTICA

APPENDIX K – Free and Informed Assent Form

ULBRA
UNIVERSIDADE LUTERANA DO BRASIL

FREE AND INFORMED ASSENT FORM
(TO MINORS FROM 12 TO 18 YEARS OLD – Resolution 466/12)

We invite you, after obtaining permission from your parents or legal guardians, to participate as a volunteer in the research: "General Relativity Theory in High School: A Study of Students' Mental Representations". This research is the responsibility of researcher Maira Giovana de Souza (Av. Farrapos 8001, building 14 - room 338, maira.souza@rede.ulbra.br) and is under the supervision of Professor PhD Agostinho Serrano de Andrade Neto (agostinho.serrano@ulbra.br).

This Consent Form may contain information you may not understand. If you have any questions, ask the person conducting the interview to clarify your participation in the research. You will not incur any costs or receive any payment for participating. You will be informed about any aspect you wish to know and will be free to participate or decline. After reading the information below, if you accept to participate in the study, sign at the end of this document, which is in two copies. One copy is for your parents to keep and the other is for the responsible researcher. If you do not accept to participate, there will be no problem, it is your right. To participate in this study, the person responsible for you must authorize and sign a Consent Form, which can be withdrawn or interrupted at any time without any prejudice.

RESEARCH INFORMATION: The research aims to investigate the mental representations developed by high school students when transitioning between Special and General Relativity theories. Students will be interviewed by the researcher. The study will take place in the second semester of 2022 during regular physics classes. The data collected through this investigation will be used in the researcher's Ph.D. thesis, which aims to investigate how mental representations are constructed and what concepts students have. The development of this research (application of research instruments) is the responsibility of the researcher, who will be available for any clarifications. I emphasize my commitment to protect the confidentiality of the information provided, which will be used exclusively for data analysis. The interviews will be recorded on video, solely for data analysis purposes, without disclosure of image and/or voice.

The information from this research will be confidential and will only be disclosed in scientific events or publications, without identifying the volunteers, except among the study's responsible parties, ensuring the secrecy of their participation. The data collected in this research (recordings, interviews, photos, etc.), will be stored in personal computer files, under the responsibility of the researcher and the supervisor, at the address provided above, for a minimum period of 5 years. Neither you nor your parents (or legal guardians) will pay anything for your participation in this research. If necessary, the expenses for your participation and those of your parents will be assumed or reimbursed by the researchers. It is also guaranteed that compensation will be provided in cases of damage caused by your participation in the research, as determined by a judicial or extrajudicial decision.

This document has been approved by the Human Research Ethics Committee of UFPE, located at: Avenida Farroupilha nº 8001 - building 14, room 224 - São José - Canoas/RS, CEP: 92425-900, Tel.: (51) 3477-9217 - email: comitedeetica@ulbra.br.

Researcher's signature: ____

MINOR'S ASSENT TO PARTICIPATE AS A VOLUNTEER

I, _____________________________________, holder of identification document ____________________ (if already issued), below signed, agree to participate in the study "General Relativity Theory in High School: A Study of Students' Mental Representations" as a volunteer. I was informed and clarified by the researcher about the research, what will be done, as well as the possible risks and benefits that may occur with my participation. I was assured that I can withdraw from participation at any time, without having to pay anything.

Location and date: ____________________ Minor's signature:_____________________________

We witnessed the request for consent, explanations about the research, and acceptance of the volunteer to participate. 02 witnesses (not related to the research team):

Name: _____________________________ Name: _____________________________
Signature: __________________________ Signature: __________________________

APPENDIX L – Free and Informed Consent Form for Minors

1. RESEARCH PROJECT IDENTIFICATION													
Project's Title: General Relativity at High School: a students' mental representations study													
Knowledge field: Science and Mathematics							Number of participants: 150				Total: 150		
Course: Doctorate in Science and Mathematics Teaching							Unit: Science and mathematics Teaching Graduate Program (PPGECIM)						
Multicentric project		Yes	x	No	x	National		International	Overseas cooperation		Yes	x	No
Research sponsor: Researcher													
Institution where the research will be developed: E. E. T. São João Batista													
Researchers' names: Maira Giovana de Souza (researcher)													

Your child (and/or minor under your care) is being invited to participate in the research project identified above. The document below contains all the necessary information about the research we are conducting. Your permission for your child to participate in this study will be of great importance to us, but if you withdraw your permission at any time, this will not cause any harm to you.

2. RESEARCH PARTICIPANT AND/OR GUARDIAN IDENTIFICATION				
Minor's name:			**Birth date:**	**Sex:**
Nationality:		**Civil Status:**	**Profession:**	
RG:	**CPF/MF:**	**Telephone:**	**Email:**	
Address:				

3. RESPONSIBLE RESEARCH'S IDENTIFICATION		
Name: Maira Giovana de Souza		Telephone: (51) 995777321
Profession: Teacher	Council register Nº:	Email: maira.souza@rede.ulbra.br
Address: Rua João Pessoa, 1468, Bairro Centro – Montenegro.		

I, the person responsible for the minor identified above, after receiving information and clarification about this research project, freely and voluntarily authorize his/her participation as a volunteer and I am aware of:

1. The justification and objectives for carrying out this research.

We believe that this research will provide students with the opportunity to understand a little more about Science, and specifically Physics, during classes. These classes will be held weekly with the researcher during the Physics curricular classes. In the classes, we will teach Physics content through various activities. With the participation of the students, we intend to investigate how they develop mental representations to understand the phenomena studied. We believe that students who participate in this project will be able to understand scientific concepts relevant to their formation as citizens.

2. The purpose of my child's participation.

The participation of your child (and/or minor under your care) is extremely important for our research, as we seek to investigate how students develop mental representations to understand the phenomena that will be studied.

3. The data collection procedure.

We will administer questionnaires to the students and, after the questionnaires are administered, we will conduct interviews with the students at times to be arranged, at the school. The interviews will be recorded on video, for the sole purpose of data analysis, without the disclosure of images and/or voices.

4. The use, storage and disposal of samples.

The data collected through this investigation will be stored by the researcher on her personal computer.

5. Discomfort and risks.

We believe that all research can involve risks. However, in this research, students will be invited to participate freely. The only risks we believe are possible are that the students may feel uncomfortable at some points during the interview, but they are free to not participate at any time, and the possibility of losing confidentiality of the data, since it will be stored on a personal computer.

6. Benefits.

By participating in this research, students will have the opportunity to learn more and better understand the phenomena studied. It will be beneficial for society and science to investigate how students develop mental representations in order to understand these phenomena.

7. Alternative methods.

We will not use alternative methods.

8. Exemption and reimbursement of expenses.

The participant will be exempt from any expenses and will not receive payment for the activity.

9. The form of monitoring and assistance.

The researcher is responsible for developing the research with the students, and I am available for any possible clarifications.

10. The freedom to refuse, withdraw or withdraw my consent.

Your child (and/or minor under your care) is free to refuse, withdraw or stop collaborating in this research at any time they wish, without the need for any explanation. Withdrawal will not cause any harm and will not interfere with the research The Theory of General Relativity in High School: a study on students' mental representations.

11. Guarantee of confidentiality and privacy.

The results obtained during this study will be kept confidential, but I agree that they may be published in scientific publications, as long as personal data is not mentioned.

12. Guarantee of clarification and information at any time.

I am guaranteed to be aware of and obtain information, at any time, about the procedures and methods used in this study, as well as the final results of this research. To this end, I may consult the researcher in charge, Maira Giovana de Souza, by email at maira.souza@rede.ulbra.br or by phone (51) 99577-7321. In case of doubts not adequately clarified by the researcher, disagreement with the procedures, or irregularities of an ethical nature, I may also contact the **Ethics Committee for Research on Human Beings of ULBRA Canoas (RS)**, located at Rua Farroupilha, 8001 – Building 14 – Room 224, Bairro São José, CEP 92425-900 - phone (51) 3477-9217, email comitedeetica@ulbra.br.

I declare that I have obtained all the necessary information and clarification regarding the doubts I have presented and, in agreement, I sign this document in two copies of the same content and form, one of which remains in my possession.

______________ (   ), _____ of _____________ of ______.

_______________________________________ _______________________________________

Research Participant **Research Participant's Guardian**

____

Researcher Responsible for the Project

APPENDIX M – Free and Informed Consent Form

<table>
<tr><td colspan="14">1. RESEARCH PROJECT IDENTIFICATION</td></tr>
<tr><td colspan="14">Project's Title: General Relativity at High School: a students' mental representations study</td></tr>
<tr><td colspan="7">Knowledge field: Science and Mathematics</td><td colspan="4">Number of participants: 150</td><td colspan="3">Total: 150</td></tr>
<tr><td colspan="7">Course: Doctorate in Science and Mathematics Teaching</td><td colspan="7">Unit: Science and mathematics Teaching Graduate Program (PPGECIM)</td></tr>
<tr><td>Multicentric project</td><td></td><td>Yes</td><td>x</td><td>No</td><td>x</td><td>National</td><td></td><td>International</td><td>Overseas cooperation</td><td></td><td>Yes</td><td>x</td><td>No</td></tr>
<tr><td colspan="14">Research sponsor: Researcher</td></tr>
<tr><td colspan="14">Institution where the research will be developed: E. E. T. São João Batista</td></tr>
<tr><td colspan="14">Researchers' names: Maira Giovana de Souza (researcher)</td></tr>
</table>

You are being invited to participate in the research project identified above. The document below contains all the necessary information about the research we are conducting. Your agreement to participate in this study will be of great importance to us, but if you withdraw your permission at any time, this will not cause any harm to you.

<table>
<tr><td colspan="5">2. RESEARCH PARTICIPANT IDENTIFICATION</td></tr>
<tr><td colspan="3">Minor's name:</td><td>Birth date:</td><td>Sex:</td></tr>
<tr><td colspan="2">Nationality:</td><td>Civil Status:</td><td colspan="2">Profession:</td></tr>
<tr><td>RG:</td><td>CPF/MF:</td><td>Telephone:</td><td colspan="2">Email:</td></tr>
<tr><td colspan="5">Address:</td></tr>
</table>

<table>
<tr><td colspan="3">3. RESPONSIBLE RESEARCH'S IDENTIFICATION</td></tr>
<tr><td colspan="2">Name: Maira Giovana de Souza</td><td>Telephone: (51) 995777321</td></tr>
<tr><td>Profession: Teacher</td><td>Council register Nº:</td><td>Email: maira.souza@rede.ulbra.br</td></tr>
<tr><td colspan="3">Address: Rua João Pessoa, 1468, Bairro Centro – Montenegro.</td></tr>
</table>

I, participant of the research identified above, after receiving information and clarification about this research project, freely and voluntarily authorize his/her participation as a volunteer and I am aware of:

1. The justification and objectives for carrying out this research.

We believe that this research will provide students with the opportunity to understand a little more about Science, and specifically Physics, during classes. These classes will be held weekly with the researcher during the Physics curricular classes. In the classes, we will teach Physics content through various activities. With the participation of the students, we intend to investigate how they develop mental representations to understand the phenomena studied. We believe that students who participate in this project will be able to understand scientific concepts relevant to their formation as citizens.

2. The purpose of my child's participation.

Your participation is extremely important for our research, as we seek to investigate how students develop mental representations to understand the phenomena that will be studied.

3. The data collection procedure.

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We will not use alternative methods.

8. Exemption and reimbursement of expenses.

The participant will be exempt from any expenses and will not receive payment for the activity.

9. The form of monitoring and assistance.

The researcher is responsible for developing the research with the students, and I am available for any possible clarifications.

10. The freedom to refuse, withdraw or withdraw my consent.

You are free to refuse, withdraw or stop collaborating in this research at any time you wish, without the need for any explanation. Withdrawal will not cause any harm and will not interfere with the research The Theory of General Relativity in High School: a study on students' mental representations.

11. Guarantee of confidentiality and privacy.

The results obtained during this study will be kept confidential, but I agree that they may be published in scientific publications, as long as personal data is not mentioned.

12. Guarantee of clarification and information at any time.

I am guaranteed to be aware of and obtain information, at any time, about the procedures and methods used in this study, as well as the final results of this research. To this end, I may consult the researcher in charge, Maira Giovana de Souza, by email at maira.souza@rede.ulbra.br or by phone (51) 99577-7321. In case of doubts not adequately clarified by the researcher, disagreement with the procedures, or irregularities of an ethical nature, I may also contact the **Ethics Committee for Research on Human Beings of ULBRA Canoas (RS)**, located at Rua Farroupilha, 8001 – Building 14 – Room 224, Bairro São José, CEP 92425-900 - phone (51) 3477-9217, email comitedeetica@ulbra.br.

I declare that I have obtained all the necessary information and clarification regarding the doubts I have presented and, in agreement, I sign this document in two copies of the same content and form, one of which remains in my possession.

_____________ (   ), _____ of ____________ of ______.

______________________________	______________________________
Researcher Responsible for the Project	**Research Participant**

APPENDIX N – Final version of slides presentation used in classes

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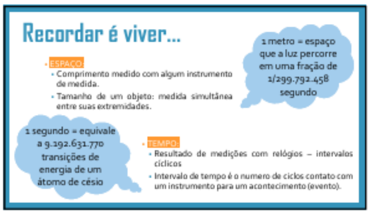

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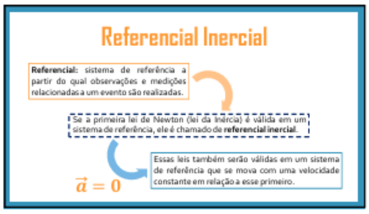

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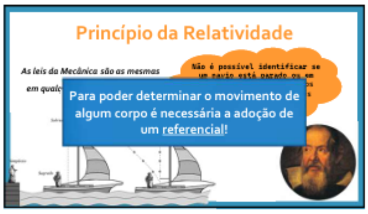

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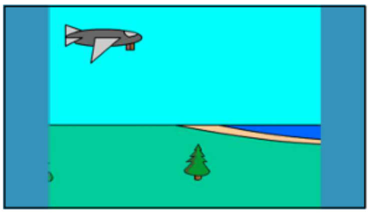

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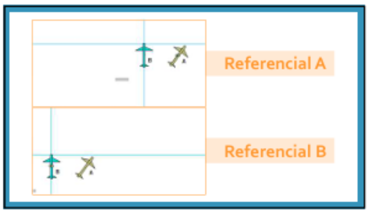

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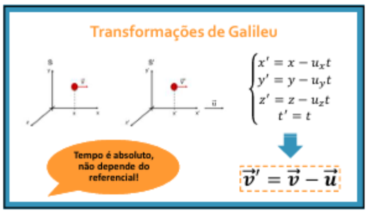

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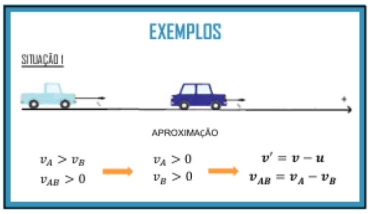

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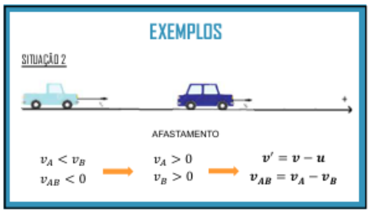

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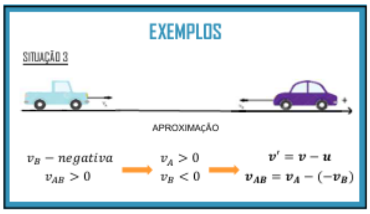

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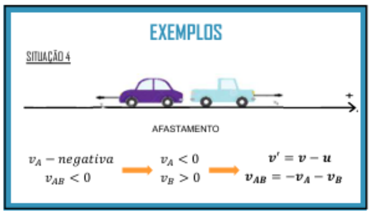

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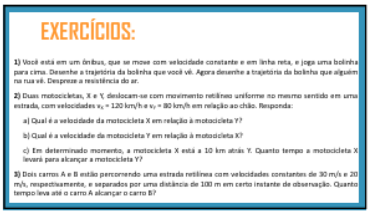

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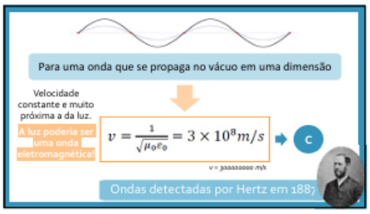

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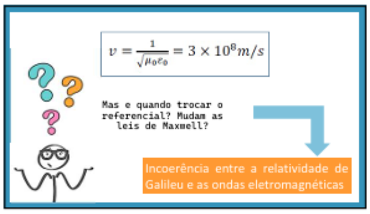

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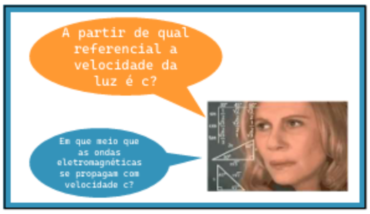

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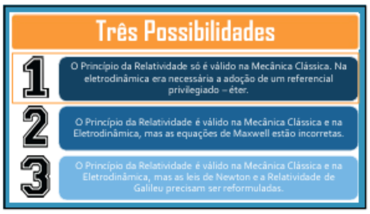

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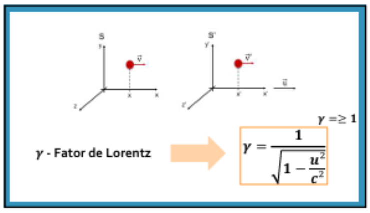

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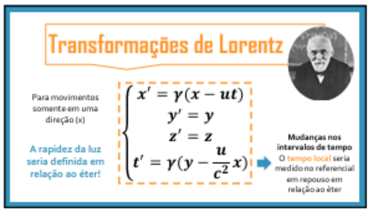

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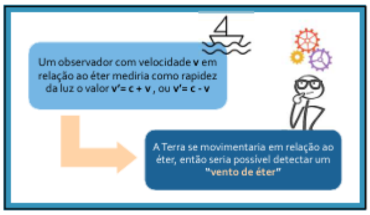

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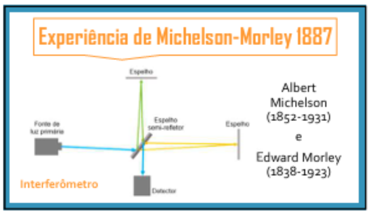

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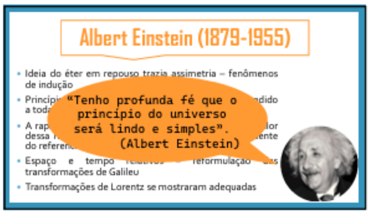

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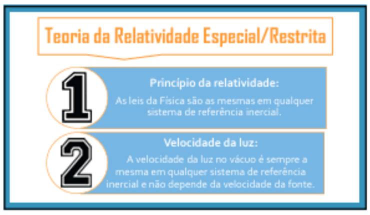

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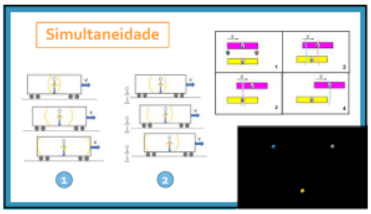

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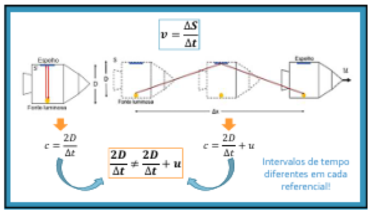

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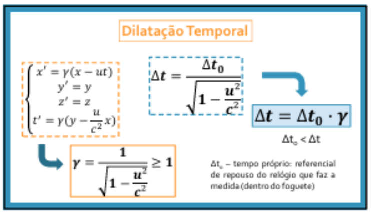

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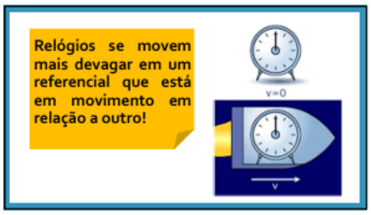

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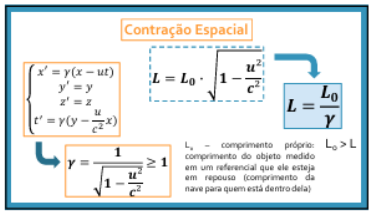

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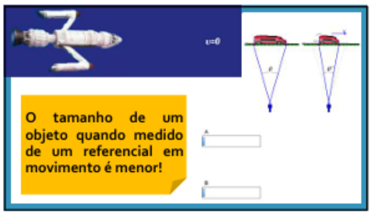

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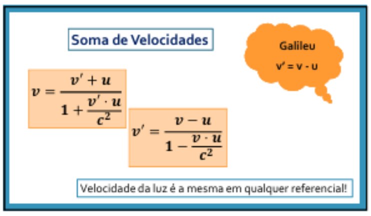

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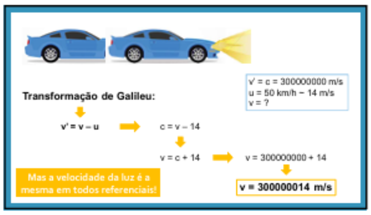

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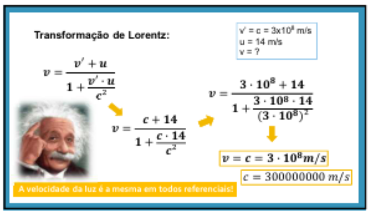

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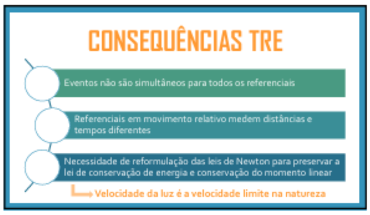

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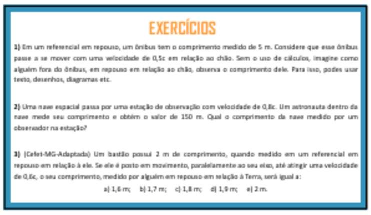

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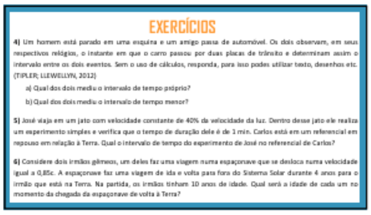

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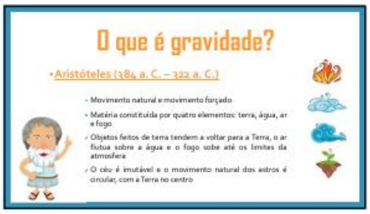

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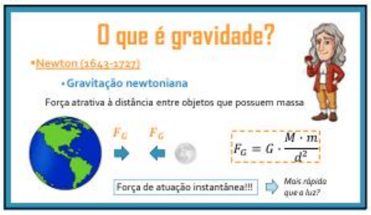

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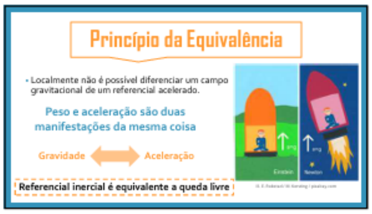

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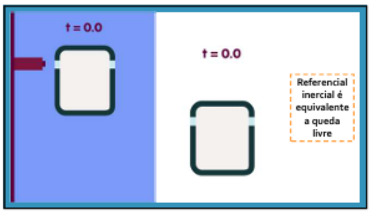

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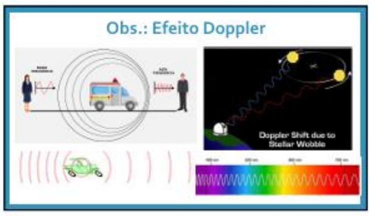

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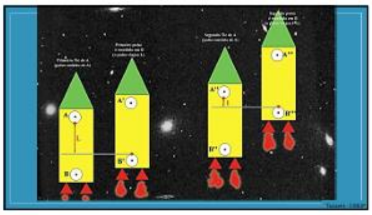

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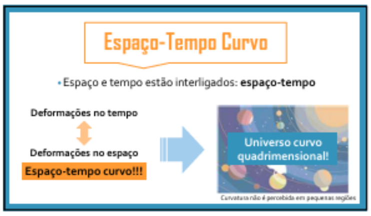

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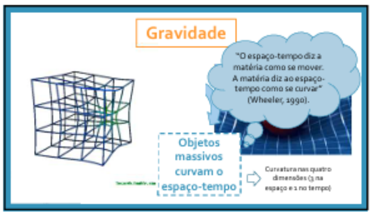

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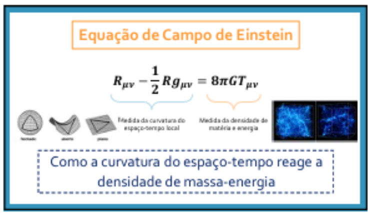

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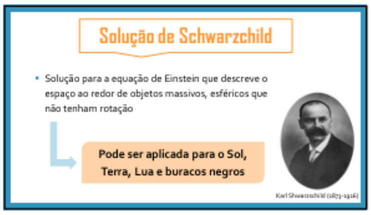

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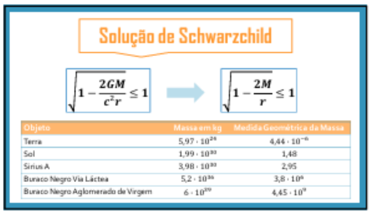

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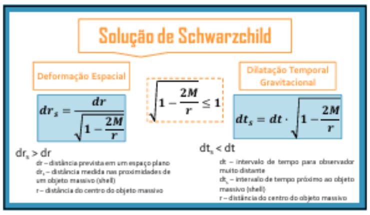

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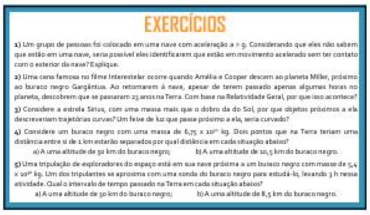

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ANNEXES

ANNEX A – Cohort 1 Activities[9]

The development of research activities began in the third quarter of 2019, during the Physics curricular. Students had previously studied topics such as Electrostatics, Electrodynamics, and Electromagnetism. Among the five classes that participated in the research, one was unable to complete the activities and another did not perform the activities using computational simulations.

Prior to the beginning of the activities, each class answered the pre-tests, using questionnaires on Galilean Transformations and Lorentz Transformations. After the tests, the content was presented through expository-dialogue lectures using a slides presentation developed through a didactic sequence. According to Zabala (1998), didactic sequences are:

> [...] a set of ordered, structured, and articulated activities for the achievement of certain educational objectives, which have a known beginning and end, both for teachers and students (ZABALA, 1998, p. 18).

Therefore, through this ordered sequence of activities, an attempt was made to bring a historical rescue, highlighting the problems that emerged historically and the need for the development of a theory that would resolve them, something that was not feasible with the prevailing theories at the time. Thus, students were able to observe a situation of model and theory exchange within Physics.

After a brief introduction to Einstein's Theory of Relativity, Galilean Relativity was worked on. Videos, GIFs, and animations from the slides were used, as well as a demonstration of the Galilean Transformations model by the teacher. Then, exercises were performed and corrected. These activities lasted for two 50-minute classes.

Next, the two computational simulations related to Galilean Transformations, called "Ball on the Train" and "Cars and Plane", were used. Since the school's available computers were not compatible with the software used, some students brought their laptops and the activities were performed in groups of around four members each. Each group received a usage guide for the simulations. This activity was performed in two 50-minute classes.

[9] Master's dissertation methodology section: DE SOUZA, M. G. Das Transformações de Galileu a Lorentz: Compreendendo as simulações mentais e concepções de estudantes do Ensino Médio sobre Relatividade Especial. Orientador: SERRANO, A. 2021. 188 f. Dissertação (Mestre em Ensino de Ciências e Matemática) - Programa de Pós-Graduação em Ensino de Ciências e Matemática, Universidade Luterana do Brasil, Canoas, RS. Disponível em: http://www.ppgecim.ulbra.br/teses/index.php/ppgecim/article/view/381/0.

Continuing the activities, more historical events were presented and discussed, culminating in the development of Special Relativity. Among the events, the measurement of light speed and the development of Maxwell's Equations were addressed, as well as the inconsistencies that emerged between them and Galilean Transformations.

Three possible solutions to the impasse were presented: that the Principle of Relativity only applied to Classical Mechanics, adopting a privileged reference frame in Electromagnetism (ether); that Maxwell's Equations were incorrect; or that Galilean Relativity and Newton's Laws needed to be reformulated.

Thus, Lorentz Transformations were introduced, as well as his view on the ether, i.e., historically, the first option was followed. The Michelson-Morley experiment was then presented, using a video representation contained in the slides, and the expected and obtained results were discussed. These activities were performed in two additional 50-minute classes.

Finally, Einstein's Special Theory of Relativity was addressed. Its two postulates and consequences were presented, focusing on the phenomena of time dilation and space contraction. It was also shown how this theory could solve the problems that had emerged, and that Lorentz Transformations were compatible with the theory, but through another interpretation.

Videos and GIFs from the slide presentation were used, as well as some situations that could only be explained by SR. Then, exercises were performed and corrected, using two additional 50-minute classes.

The last part of the in-class activities was the use of simulations on Lorentz Transformations for Special Relativity, "Time Dilation" and "Space Contraction". As in the Galilean simulations' activity, some students brought their laptops again and the activity was performed in groups of around four members each. Here, too, usage guides were provided for each group. Two additional 50-minute classes were used.

Finally, the two post-tests were administered, one for each situation. The same questionnaires used as the pre-tests were used to compare the responses of each student before and after the activities. The questionnaires from each student were analysed and classified into four categories based on their responses. Through them, 14 students were selected for interviews, which will be described below.

ANNEX B – 3rd year physics study plan of São João Batista State Technical School

Syllabus	Interpretation and understanding of the concepts and laws of nature that enable the knowledge of phenomena affecting life on Earth and serve as a basis for understanding contemporary technologies, such as the concepts and laws of classical mechanics, fluid mechanics, wave mechanics, thermodynamics, optics, and electromagnetism. Study of the impact of technologies associated with natural sciences on one's personal life, production processes, knowledge development, and life, as well as the application of these technologies in school, work, and other relevant contexts for one's life.
3rd YEAR	
First Quarter	Electrostatics.
Second Quarter	Electrodynamics. Magnetism.
Third Quarter	Electromagnetism. Modern Physics.
Methodology	Theoretical classes - expository and experimental, technological application debates. Practical activities, portfolios, graphs, scientific research.
Assessment	It is a continuous, participatory, and investigative process. The goal is to diagnose advancements and deficiencies, in order to intervene and act.